Institutional Investors and Information Acquisition: Implications for Asset Prices and Informational Efficiency*

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Abstract

We study the joint portfolio and information choice problem of institutional investors who are concerned about their performance relative to a benchmark. Benchmarking influences information choices through two distinct economic mechanisms. First, benchmarking reduces the number of shares in investors' portfolios that are sensitive to information. Hence, the value of private information declines. Second, benchmarking limits investors' willingness to speculate. While this reduces the value of private information as well, importantly it also adversely affects information aggregation. In equilibrium, investors acquire less information and informational efficiency declines. As a result, return volatility increases, and less-benchmarked institutional investors outperform more-benchmarked ones.

Keywords: benchmarking, institutional investors, informational efficiency, asset allocation, asset pricing

JEL: G11, G14, G23

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Institutional investors own a majority of U.S. equity and account for most of the transactions and trading volume in financial markets.[1] Notably, the importance of relative performance concerns for institutional investors has grown steadily over the past years. That is, the performance of many institutions is nowadays evaluated relative to a benchmark portfolio ("benchmarking"), either explicitly, for example, through performance fees,[2] or implicitly, for instance, due to the flow-performance relation.

There is now a growing body of literature that studies the asset pricing implications of benchmarking for the case of symmetrically informed institutional investors. In contrast, the focus of this paper is on the impact of benchmarking on information acquisition and informational efficiency. In particular, our objective is to understand the economic mechanisms through which the growth of assets under management by benchmarked institutions affects informational efficiency and asset prices.

For that purpose, we develop an equilibrium model of *joint* portfolio and information choice that explicitly accounts for relative performance concerns. The two key features of the model are (i) that all institutional investors endogenously decide on the precision of their private information, and (ii) that a fraction of institutional investors ("benchmarked investors") are concerned about their performance relative to a benchmark. Otherwise the framework is kept as simple as possible to illustrate the economic mechanisms in the clearest possible way; for example, for most of the analysis we focus on the case in which investors can trade a risk-free bond and a single risky stock. Varying the fraction of benchmarked investors, or, equivalently, the share of assets managed by benchmarked institutions, will be our key comparative statics analysis to illustrate the implications of benchmarking.

We identify two distinct economic channels through which benchmarking influences investors' information choices: (i) *information-scale effects* and (ii) *risk-taking effects*. Both mechanisms are driven by the *interaction* between portfolio and information choice. We

[1]For evidence on institutional ownership and trading volume, see French (2008), U.S. Securities and Exchange Commission (2013), Stambaugh (2014), and Griffin, Harris, and Topaloglu (2003), respectively.

[2]For example, recently, many fund managers have introduced new performance-linked fee structures, including AllianceBernstein, Allianz Global Investors, Equitile, Fidelity International and Orbis Investment Management. Also, Japan's Government Pension Investment Fund, the world's largest retirement fund, introduced a system whereby it pays all active managers a fee based on their return relative to a benchmark. Moreover, individual portfolio manager's compensation is also often performance-based (see Ma, Tang, and Gomez (2018)).

illustrate these two effects by means of two economic settings, the first one of which keeps, by design, investors' risk appetite and, hence, aggregate risk-bearing capacity fixed.

In particular, we illustrate information-scale effects arising from benchmarking in a tractable model in which institutional investors have constant absolute risk-aversion utility but a preference for the early resolution of uncertainty.[3] We show that benchmarked investors' portfolios can be decomposed into two components. First, the standard mean-variance portfolio, that is, the optimal portfolio of non-benchmarked investors. This component is not affected by benchmarking but rather driven by investors' posterior beliefs. Second, a hedging portfolio arising from benchmarking. Intuitively, because benchmarked institutional investors strive to do well when the benchmark performs well, they over-weight the benchmark portfolio. This hedging portfolio is designed to track the benchmark, not to outperform it, and, consequently, it is information-insensitive.

Importantly, conditional on investors' information choices, these investment decisions do not affect the sensitivity of the stock price with respect to the payoff or the noise. Hence, benchmarking does not adversely affect information aggregation. Instead, benchmarking reduces the *value* of information and, thus, investors' incentives to acquire private information. Intuitively, once the stock market clears, the aggregate—information-insensitive—hedging demand of the benchmarked institutions reduces the "effective supply" of the stock, that is, the number of shares in the economy that are available for speculation.[4] As a result, for all—benchmarked and non-benchmarked—investors, the expected number of shares in their portfolio that are sensitive to information declines.[5] Because private information is then applied to fewer shares, its marginal value is lower. With no change in information costs, investors acquire less private information. Consequently, price informativeness declines in the fraction of benchmarked investors in the economy. This, in turn, implies a higher return volatility since the price tracks the stock's payoff less closely.

[3]These preferences are chosen for illustration only. In the case of constant relative risk-aversion preferences, as discussed next, information-scale effects are also present. But, the model is less tractable and benchmarking affects the aggregate risk-bearing capacity as well.

[4]The same effect would arise in the presence of "index investors," i.e., investors whose portfolio choice is not driven by (private) information.

[5]This holds for the economically relevant case in which the effective supply is positive which guarantees a positive risk premium for the stock.

The implications described thus far follow from scale effects in information *acquisition*. In particular, the decline in price informativeness is entirely determined by the decline in the investors' precision choices. However, benchmarking also affects information *aggregation* through risk-taking effects, which we illustrate using a model with constant relative risk-aversion preferences.[6]

Benchmarked investors' portfolios can be (approximately) decomposed into the same two components as in the mean-variance setting. The key difference is that relative performance concerns now also affect the mean-variance portfolio. In particular, benchmarking limits institutional investors' willingness to speculate.[7] Hence, benchmarked investors not only acquire less information but also trade less aggressively on a particular piece of information and, thus, benchmarking adversely affects information aggregation. This amplifies the decline in price informativeness.

Risk-taking effects have important implications. For example, in contrast to the case of mean-variance preferences, the information choices of benchmarked and non-benchmarked institutional investors differ. In particular, benchmarked investors choose a lower precision of private information. As a result, benchmarked investors are less well informed and, hence, earn lower expected portfolio returns than non-benchmarked investors. Moreover, as the assets under management of benchmarked institutions increase, less information is revealed through the public stock price, such that the "information gap" and, hence, the return gap between investors widen. For realistic calibrations, some of the asset-pricing implications change even qualitatively. For example, the price of the stock can decline in the fraction of benchmarked investors, or, equivalently, its expected excess return can increase.

Finally, we study two extensions of our basic economic framework. First, we extend our model to multiple stocks and document that informational efficiency also deteriorates for stocks that are not part of the benchmark, although to a smaller extent because the information-scale effect is absent for non-index stocks. Second, we study benchmarking concerns that are nonlinear in the benchmark's performance—in line with asymmetric per-

[6] Because the equilibrium price function in this model is nonlinear, it is considerably less tractable and we rely on a novel numerical solution method to solve it.

[7] Technically, benchmarking increases the local coefficient of relative risk-aversion because proportional movements in an investor's wealth have a larger impact in the presence of relative performance concerns.

formance fee structures and implicit in the flow-performance relationship. We demonstrate that asymmetric benchmarking concerns can mitigate the adverse effects of benchmarking on information choice and information aggregation.

We also make a methodological contribution by developing a novel numerical solution approach that allows us to determine the equilibrium in noisy rational expectations models with nonlinear price functions. Our approach differs from previous attempts, such as Bernardo and Judd (2000), in that it does not rely on a parameterization of the price law and the asset demand nor on the projection method. Our solution technique is flexible and could also be used to study rational expectation equilibrium models in the presence of constraints or frictions.

The two papers that are closest to our work are Admati and Pfleiderer (1997) and García and Strobl (2011), whose results and implications, however, are distinctly different. Admati and Pfleiderer (1997) study linear benchmarking concerns in the compensation of privately informed portfolio managers, but in a framework with CARA preferences and an exogenous price process. They document that, because each manager can use his portfolio choice to "undo" the benchmarking component in his compensation, the manager's optimal information choice is not affected by relative performance concerns. In contrast, taking into account the effect of investors' portfolio choices on the market-clearing stock price, we illustrate that benchmarking leads to a decline in information acquisition. We also document a novel effect resulting from benchmarked investors' risk-taking.

García and Strobl (2011) study how relative wealth concerns affect investors' incentives to acquire information. They demonstrate that, when an investor's utility depends on the consumption of the average investor, complementarities in information acquisition arise, introducing the possibility of multiple "herding" equilibria. These complementarities can lead to an increase in informed trading, thereby improving price informativeness. The key difference is that, in our framework, investors have relative performance concerns (with respect to a benchmark), and not relative wealth concerns (with respect to other investors). As a result, in our setting, no complementarities in information choice arise and price informativeness declines.

Kacperczyk, Nosal, and Sundaresan (2018) develop a rational expectations model with market power and demonstrate that active investors shift their information choice in reaction to an increase in the size of passive investors. As a result, price informativeness declines. While passive investors in their model are, by definition, uninformed, the benchmarked investors in our framework endogenously choose the precision of their private information.

The economic framework that we develop builds on two independent strands of research: first, the asset pricing literature on the stock market implications of benchmarking (absent information choice). Cuoco and Kaniel (2011) and Basak and Pavlova (2013) study models with CRRA preferences and relative performance concerns. They document that, in the presence of benchmarking, institutional investors optimally tilt their portfolio toward the benchmark, creating upward price pressure.[8] Buffa, Vayanos, and Woolley (2017) study a setup with CARA preferences in which, due to agency frictions, asset-management contracts endogenously depend on fund managers' performance relative to a benchmark.

Unlike these papers, we explicitly model investors' joint information and portfolio choice in the presence of benchmarking. Notably, allowing for endogenous information choice can lead to qualitatively different asset pricing implications. For example, stock prices can decline in the fraction of benchmarked investors. We also provide novel predictions regarding institutional investors' expected portfolio returns.

Second, our framework builds on the literature on information acquisition in competitive markets (absent benchmarking).[9] Information-scale effects are discussed in van Nieuwerburgh and Veldkamp (2009, 2010), who document a feedback effect between information and portfolio choice through the number of shares investors expect to hold, and Peress (2010), who demonstrates that better risk-sharing lowers the value of information because investors expect to hold fewer shares.[10]

[8]Related, Brennan (1993) derives a two-factor model, one of which being the benchmark and Buffa and Hodor (2018) show how heterogeneous benchmarking can result in negative spillovers across assets.

[9]Our work is also related to recent studies that have relaxed the joint CARA-normal assumption (see, e.g., Barlevy and Veronesi (2000), Peress (2004), Albagli, Hellwig, and Tsyvinski (2014), Breon-Drish (2015) and Chabakauri, Yuan, and Zachariadis (2017)).

[10]Also related, García and Vanden (2009) show that competition between fund managers makes prices more informative. Malamud and Petrov (2014) demonstrate that convex compensation contracts lead to equilibrium mispricing. Kacperczyk, van Nieuwerburgh, and Veldkamp (2016) and Farboodi and Veldkamp (2017) discuss what data fund managers optimally choose to process. Bond and García (2016) study the

Unlike these papers, we explicitly model institutional investors who are concerned about their relative performance, which allows us to make novel predictions about the relationship between the size of benchmarked institutions and informational efficiency.

The remainder of the paper is organized as follows. Section 1 introduces our economic framework and discusses investors' optimization problems. Sections 2 and 3 discuss how benchmarking affects informational efficiency as well as asset prices through information-scale effects and risk-taking effects, respectively. Section 4 discusses two extensions of our basic framework. Finally, Section 5 summarizes the key predictions and concludes. Proofs and a description of the numerical solution approach are delegated to the Appendix.

1 The Model

This section introduces our basic economic framework, which explicitly accounts for the information choices of institutional investors who are concerned about their performance relative to a benchmark. In particular, we incorporate benchmarking concerns, as in Cuoco and Kaniel (2011) and Basak and Pavlova (2013), into a competetive rational expectations equilibrium model of joint portfolio and information choice, as in Verrecchia (1982). We also discuss investors' optimization problems and the equilibrium concept.

1.1 Economic Framework

Information Structure and Timing

We consider a static model, which we break up into three (sub-)periods. Figure 1 illustrates the sequence of the events. In period 1, the information choice period, investors can spend time and resources to acquire private information about a stock. For example, they may study financial statements, gather information about consumers' taste, hire outside financial advisers, or subscribe to proprietary databases. In particular, each investor i can choose the

consequences of investing solely via the market portfolio and find negative externalities for uninformed investors.

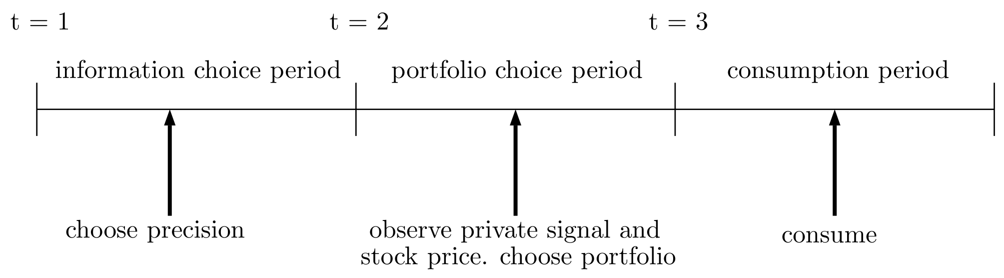

Figure 1: Timing. The figure illustrates the sequence of the events.

precision q_i of his private signal Y_i. Higher precision will reduce the posterior uncertainty regarding the stock's payoff but will increase the information acquisition costs $\kappa(q_i)$.[11]

In period 2, the portfolio choice period, investors observe their private signals (with the chosen precision) and make their investment choice. Prices are set such that markets clear. In period 3, the consumption period, investors consume the proceeds from their investments.

We denote the expectation and variance conditional on prior beliefs as $E_1[\cdot]$ and $Var_1(\cdot)$. To denote investor i's expectation and variance conditional on his time-2 information set $\mathcal{F}_i = \{Y_i, P\}$, we use $E_2[\,\cdot\,|\,\mathcal{F}_i]$ (or, $E_2[\,\cdot\,]$) and $Var_2(\,\cdot\,|\,\mathcal{F}_i)$ (or, $Var_2(\,\cdot\,)$).

Investment Opportunities

There exist two financial securities that are traded competitively in the market: a risk-less asset (the "bond") and a risky asset (the "stock"). The bond pays an exogenous (gross) interest rate R_f and is available in perfectly elastic supply. It also serves as the numéraire, with its price being normalized to one. The stock is modeled as a claim to a random payoff X, which is only observable in period 3. Its price is denoted by P. The supply of the stock, denoted by Z, is assumed to be random and unobservable. This prevents the price from fully revealing the information acquired by the investors and, thus, preserves the incentives to acquire private information in the first place.

[11] The information cost function κ is assumed to be continuous, increasing and strictly convex, with $\kappa(0) = 0$. This guarantees the existence of interior solutions and captures the idea that each new piece of information is more costly than the previous one.

Investors

There exists a continuum of atomless investors with mass one that we separate into two groups of institutional investors: (1) a fraction Γ of *benchmarked institutions*, or, short, "benchmarked investors," $\mathcal{BI}$; and (2) a fraction $1-\Gamma$ of non-benchmarked institutions, or, short, "non-benchmarked investors," $\mathcal{NI}$. Each investor $i \in \{\mathcal{BI}, \mathcal{NI}\}$ is endowed with the same initial wealth $W_{0,i}$, which we normalize to 1.

Motivated by recent theoretical contributions,[12] we model the compensation of institutional investors, C_i, as:[13]

$$C_i(W_i, R_B) = W_i - \gamma_i \, W_{0,i} \, R_B - \kappa(q_i). \tag{1}$$

Institutional investors' compensation has two components; first, a standard component related to terminal wealth W_i (i.e., "assets under management");[14] and, second, a linear benchmarking component that is related to the performance of the benchmark R_B.[15] Consequently, γ_i captures the strength of investor i's benchmarking concerns; in particular, while benchmarked institutional investors are concerned about their performance relative to a benchmark ($\gamma_i > 0, \forall i \in \mathcal{BI}$), non-benchmarked investors are not ($\gamma_i = 0, \forall i \in \mathcal{NI}$). Finally, information acquisition costs $\kappa(q_i)$ are deducted.

It is important to highlight that benchmarking is the only source of heterogeneity across the two groups of institutional investors. In particular, benchmarking does not affect the

[12] Basak and Pavlova (2013) demonstrate benchmarking formally using an agency-based argument. In Buffa, Vayanos, and Woolley (2017), investors endogenously—due to agency frictions—make fund managers' fees sensitive to the performance of a benchmark. Similarly, Sotes-Paladino and Zapatero (2016) show that a linear benchmark-adjusted component in managers' contracts can benefit investors.

[13] This compensation scheme is chosen for simplicity and analytical tractability. In earlier versions of the paper, we had explicitly modeled the compensation of institutional investors as a function of their "out-performance," i.e., the difference between the investors' return and the benchmark return. This has, qualitatively, no effect on the results.

[14] Considering instead a *fraction* of terminal wealth, e.g., $\beta_i \, W_i$, has qualitatively no impact on the results. It affects the total amount of information that investors acquire, but not the impact of benchmarking.

[15] These types of benchmarking concerns capture linear ("Fulcrum") performance fees. The 1970 Amendment of the Investment Advisers Act of 1940 restricts mutual fund fees in the U.S. to be of the Fulcrum type. An investor's desire to perform well relative to a benchmark may also be driven by social status, instead of monetary incentives. Note also that these benchmarking concerns do not capture relative wealth concerns, these are discussed in García and Strobl (2011).

investors' financial wealth but only their utility; that is, in the portfolio choice period, both groups of institutional investors have the same capital available for investment.

We consider a general form of investors' preferences over compensation C_i:

$$U_{1,i} = E_1\left[u_1\left(\, E_2\left[u_2(C_i)\right]\,\right)\right], \tag{2}$$

in which the inner utility function u_2 governs risk-aversion and the outer utility function u_1 governs the preference for the timing of the resolution of uncertainty.[16] Specifically, if u_1 is linear, investors are indifferent about the timing, whereas a convex (concave) function u_1 implies a preference for early (late) resolution of uncertainty. In the next sections, we explore specific examples.

The specification of the benchmarked investors' utility over compensation C_i exhibits two important characteristics that let them behave differently from non-benchmarked investors. First, benchmarked investors have an incentive to post a high return when the benchmark performs well or, formally, their marginal utility is increasing in R_B. Second, benchmarked investors' utility function is decreasing in the performance of benchmark, thus affecting information choice. Note that our definition of the investors' compensation scheme in (1) shares many similarities with the specifications in Cuoco and Kaniel (2011) and Basak and Pavlova (2013).[17]

[16]Technically, u_1 and u_2 are assumed to be continuous, twice differentiable, and increasing functions.

[17]Compensation in Cuoco and Kaniel (2011) is composed of a constant "load fee," a fraction of terminal wealth, and a performance-related component. Basak and Pavlova (2013) study a tractable specification similar to ours. However, utility is increasing in the benchmark (which is less important in the absence of information choice because only marginal utility matters). In the case of $\gamma_i = 1$, our specification in (1) is comparable to the alternative specification discussed in their Remark 1.

1.2 Investors' Optimization Problems and Equilibrium

Portfolio and Information Choice

In the portfolio choice period ($t = 2$), investor i chooses the number of shares of the stock, θ_i, in order to maximize expected utility; conditional on his posterior beliefs and price P:

$$U_{2,i} = \max_{\theta_i} E_2\big[\, u_2(C_i) \,|\, \mathcal{F}_i\big], \tag{3}$$

with terminal wealth, W_i, being given by $W_i = W_{0,i}\, R_f + \theta_i\,(X - PR_f)$.[18]

In the information choice period ($t = 1$), investor i chooses the precision of his private signal, q_i, in order to maximize expected utility over all possible realizations of his private signal Y_i and the public price P, anticipating his optimal portfolio choice in period 2:

$$\max_{q_i \geq 0} E_1\big[u_1(U_{2,i})\big]. \tag{4}$$

Equilibrium Definition

A rational expectations equilibrium is defined by portfolio choices $\{\theta_i\}$, information choices $\{q_i\}$, and prices $\{P\}$ such that:

1. θ_i and q_i solve investor i's maximization problems in (3) and (4), taking P as given.
2. Expectations are rational; that is, the average precision of private information implied by aggregating investors' precision choices equals the level assumed in investors' optimization problems (3) and (4).
3. Aggregate demand equals aggregate supply.

Note that, in equilibrium, the stock price plays a dual role: It clears the security market and aggregates as well as disseminates investors' private information.

[18] This follows from the two budget equations $W_i = \theta_i\, X + \theta_i^B\, R_f$ and $W_{0,i} = \theta_i\, P + \theta_i^B$ by solving the second equation for θ_i^B (number of shares of the bond) and plugging the solution into the first.

2 Benchmarking and Information Scale Effects

In this section, we illustrate how benchmarking affects equilibrium due to *information-scale effects*. For this purpose, we rely on a tractable model that is designed to provide the economic intuition and allows for closed-form solutions of all quantities in the economy. In particular, we demonstrate that, in the presence of information-scale effects, price informativeness deteriorates. Importantly, this drop in price informativeness is *entirely* determined by a decline in the *value* of private information and, hence, a decline in investors' average precision choices, whereas information aggregation is not adversely affected.

2.1 Setup

We assume that the payoff of the stock, X, and its supply, Z, are normally distributed, with $X \sim \mathcal{N}(\mu_X, \sigma_X^2)$ and $Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2)$. The investors' private signals are given by $Y_i = X + \varepsilon_i$, with $\varepsilon_i \sim \mathcal{N}(0, 1/q_i)$. Investors have "mean-variance preferences" of the Kreps-Porteus type. Their time-1 utility function, $U_{1,i}$, is given by:[19]

$$U_{1,i} = E_1\left[E_2\left[C_i\right] - \frac{\rho}{2} Var_2\left(C_i\right)\right], \tag{5}$$

where compensation C_i is given by $C_i = W_i - \gamma_i\ (X - PR_f) - \kappa(q_i)$, with $W_{0,i}R_B \equiv X - PR_f$ capturing the performance of the benchmark. These preferences lead, in period 2, to the same portfolio that obtains under CARA expected utility, but investors have a preference for the early resolution of uncertainty.[20] They can also arise in a setting with risk-neutral, profit-maximizing portfolio managers who invest on behalf of clients with CARA expected utility (see, e.g., the discussion in footnote 10 in van Nieuwerburgh and Veldkamp (2009)).

It is important to highlight that in this setting, by design, benchmarking does not affect the benchmarked investors' risk appetite. As a result, changes in the fraction of benchmarked institutional investors, Γ, have no impact on aggregate risk-bearing capacity.

[19] This is a special case of the general form in (2), with $u_1(x) = -\ln(-x)$ and $u_2(x) = -\exp(-\rho x)$.

[20] Kreps and Porteus (1978) provide the axiomatic foundations for this class of non-expected utility, which allows to disentangle risk-aversion from the elasticity of inter-temporal substitution. The generalizations of iso-elastic utility in Epstein and Zin (1989) and Weil (1990) are widely used in asset pricing.

2.2 Portfolio Choice and the Equilibrium Price

An investor's optimization problem must be solved in two stages, starting with the optimal portfolio choice in period 2, taking information choices as given. The following theorem characterizes an investor's optimal portfolio choice for arbitrary values of his posterior mean $\hat{\mu}_{X,i} \equiv E_2[X \mid \mathcal{F}_i]$ and posterior precision $h_i \equiv Var_2(X \mid \mathcal{F}_i)^{-1}$.

Theorem 1. *Conditional on an investor's posterior beliefs, described by $\hat{\mu}_{X,i}$ and h_i, and the stock price P, the optimal stock demand equals:*

$$\theta_i = h_i \, \frac{\hat{\mu}_{X,i} - P\,R_f}{\rho} + \gamma_i \;\equiv\; \theta_i^{MV} + \gamma_i. \tag{6}$$

The optimal demand for the stock has two components. First, for all investors, the standard mean-variance portfolio, θ_i^{MV}. Second, for benchmarked investors, a hedging demand $\gamma_i > 0$. Intuitively, benchmarked investors have the desire to acquire assets that do well when the benchmark does well, or, equivalently, assets that co-vary positively with the benchmark. In our single-asset economy, the stock, also serving as the benchmark, naturally co-varies positively with the benchmark.

The mean-variance portfolio, θ_i^{MV}, is independent of the strength of an investor's benchmarking concerns, γ_i. In contrast, the hedging component is increasing in the investor's benchmarking concerns, γ_i, but is *information-insensitive*, that is, it is not affected by his posterior beliefs. Intuitively, it is designed to closely track or, formally, co-vary with, the benchmark, and not meant for speculation.

Aggregating the demand of both groups of institutional investors and imposing market-clearing delivers the equilibrium stock price:

Theorem 2. *Conditional on investors' information choices, described by q_i, $\forall i$, there exists a unique linear rational expectations equilibrium:*

$$P\,R_f = \frac{1}{\bar{h}}\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\,\bar{q}}{\sigma_Z^2\,\rho} + \rho\,\bar{\gamma}\right) + \frac{1}{\bar{h}}\left(\bar{h} - \frac{1}{\sigma_X^2}\right)X - \frac{1}{\bar{h}}\left(\frac{\bar{q}}{\rho\,\sigma_Z^2} + \rho\right)Z, \tag{7}$$

$$\textit{where} \qquad h_0 \equiv \frac{1}{\sigma_X^2} + \frac{\bar{q}^2}{\rho^2\,\sigma_Z^2}, \qquad \bar{h} \equiv h_0 + \bar{q}, \tag{8}$$

$$\bar{q} \equiv \int_0^\Gamma q_i^{\mathcal{BI}}\,di + \int_\Gamma^1 q_i^{\mathcal{NI}}\,di, \quad \textit{and} \quad \bar{\gamma} \equiv \int_0^\Gamma \gamma_i\,di. \tag{9}$$

The characterization of the equilibrium price in (7) is standard for this type of economy, and the variables defined in (8) and (9) allow for intuitive interpretations. h_0 equals the sum of the precisions from the prior, $1/\sigma_X^2$, and from the price signal, $\bar{q}^2/(\rho^2\,\sigma_Z^2)$; hence, it characterizes the precision of public information. $\bar{q}$ measures the precision of the private information of the average investor. Consequently, $\bar{h}$ governs average *aggregate* precision, that is, the precision of public and private information of the average investor. Finally, $\bar{\gamma}$ captures the degree of benchmarking in the economy, thereby aggregating the strength of the relative performance concerns of each benchmarked investor, γ_i, and their size in the economy, Γ. Note that, for ease of exposition, we assume for all graphical illustrations that the strength of the benchmarked investors' relative performance concerns coincide: $\gamma_i = \gamma, \forall i \in \mathcal{BI}$. As a result, $\bar{\gamma}$ simplifies to $\Gamma\,\gamma$ and, conditional on γ, all variation in $\bar{\gamma}$ is driven by the fraction of benchmarked institutions, Γ.

Conditional on information choices, the equilibrium price exceeds the price in an economy without benchmarking by $(\rho\,\bar{\gamma})/\bar{h}$; hence, the price is increasing in the degree of benchmarking $\bar{\gamma}$. Intuitively, conditional on a given supply Z, the excess demand for the stock resulting from benchmarked investors' hedging demand, drives up the price.[21]

It is important to highlight that *information aggregation is not adversely affected* by benchmarking in this setting. In particular, one can derive the following corollary that

[21] Technically, $\partial(PR_f)/\partial\bar{\gamma} = \rho/\bar{h}$. Because the magnitude of an individual investor's hedging demand relative to his mean-variance portfolio is increasing (decreasing) in risk-aversion (aggregate posterior precision), the sensitivity of the price with respect to benchmarking is positively (negatively) related to ρ ($\bar{h}$).

describes the ability of financial markets to aggregate private information in the presence of benchmarking:

Corollary 1. *Conditional on investors' information choices, described by q_i, $\forall i$, price informativeness, defined as the precision of the public price signal, is given by:*

$$h_0 - \frac{1}{\sigma_X^2} = \frac{\bar{q}^2}{\rho^2\,\sigma_Z^2}. \tag{10}$$

Specifically, equation (10) implies that, conditional on investors' information choices (as aggregated by $\bar{q}$), price informativeness is not affected by $\bar{\gamma}$. Thus, given information choices, a shift in the degree of benchmarking in the economy has no impact on how much information is revealed by the public stock price or how much investors can learn from the price. Intuitively, because the aggregate hedging demand is fully predictable, it only affects the level of the stock price but not its sensitivity with respect to the payoff or the noise.[22]

2.3 Information Choice

While Theorem 2 and Corollary 1 take the information environment as given, information choices are actually an endogenous outcome of the model. At time $t = 1$, each investor i chooses the precision of his private signal, q_i, in order to maximize utility (5), anticipating his optimal portfolio choice in period 2. In order to impose rational expectations, we substitute the optimal stock demand (6) into wealth W_i and, hence, compensation, C_i, which yields the following time-1 objective function:

$$U_{1,i} = R_f \left(W_{0,i} - \frac{\kappa(q_i)}{R_f} \right) + \frac{1}{2\,\rho}\, E_1\left[z_i^2\right], \tag{11}$$

where $z_i \equiv \sqrt{h_i}\,(\hat{\mu}_{X,i} - P\,R_f)$ denotes the investor's time-2 expected Sharpe ratio—a function of Y_i, q_i and P. Intuitively, $E_1\left[z_i^2\right]$ governs the squared Sharpe ratio that investor i expects to achieve in the information choice period ($t = 1$).

[22]Accordingly, the signal-to-noise ratio, defined as the ratio of the price sensitivities with respect to the payoff and the noise, given by $-\bar{q}/\rho$, is also unaffected by $\bar{\gamma}$.

The objective function captures the "information choice trade-off." Higher precision q_i leads to higher information acquisition costs $\kappa(q_i)$, thereby reducing time-1 utility. On the other hand, higher precision q_i increases the posterior precision h_i, the time-1 expected squared Sharpe ratio $E_1[z_i^2]$ and, thus, time-1 utility.

Notably, benchmarked investors ($i \in \mathcal{BI}$) "undo" the benchmarking component in their compensation; that is, a benchmarked investor's time-1 objective function does not depend on the strength of his *individual* benchmarking concerns, γ_i and, hence, coincides with the objective function of a non-benchmarked investor. As a result, the two groups of institutional investors actually face the same information choice problem. Accordingly, in the following, we discuss the information choice problem of a generic institutional investor $i \in \{\mathcal{BI}, \mathcal{NI}\}$, noting that $q_i = q_i^{\mathcal{BI}} = q_i^{\mathcal{NI}}$.

Computing the first-order condition of $U_{1,i}$ with respect to q_i, delivers an investor's best information choice, given arbitrary precision choices by the other investors:

Theorem 3. *Conditional on the average private signal precision, $\bar{q}$, investor i's optimal signal precision $q_i(\bar{q})$ is the unique solution of:*

$$2\,\kappa'(q_i) = \frac{1}{\rho} \underbrace{\left[\frac{1}{\bar{h}^2}\left(\rho^2\left(\sigma_Z^2 + (\mu_Z - \bar{\gamma})^2\right) + \bar{h} + \bar{q}\right)\right]}_{\equiv A(\bar{\gamma},\bar{q})}. \tag{12}$$

If $\mu_Z - \bar{\gamma} > 0$, an investor's best information choice $q_i(\bar{q})$ shifts down when the degree of benchmarking in the economy, $\bar{\gamma}$, increases. Formally, $\partial A/\partial\bar{\gamma} < 0$.

Equation (12) characterizes that, at the optimum, the marginal cost of private information, $\kappa'(q_i)$, equals the marginal benefit which is governed by $A(\bar{\gamma}, \bar{q})$ and risk aversion ρ. Hence, the degree of benchmarking $\bar{\gamma}$ affects the marginal value of private information. In particular, if the "effective supply" of the stock, that is, the supply after accounting for the aggregate hedging demand is positive ($\mu_Z - \bar{\gamma} > 0$),[23] the value of private information declines in the degree of benchmarking—for benchmarked and non-benchmarked investors.

[23] This is the economically interesting case because it guarantees that the stock's risk premium is positive (confer (A16) in Appendix A).

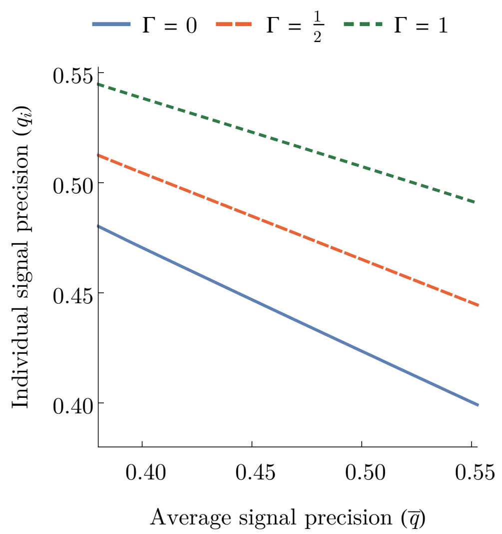

Figure 2: Information demand. The figure depicts an investor's optimal signal precision q_i as a function of the average private signal precision $\bar{q}$—for different degrees of benchmarking, as captured by the fraction of benchmarked investors, Γ. The graph is based on the framework described in Section 2.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = 1.0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BM}$, $\bar{q} = 0.45$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

This effect is illustrated in Figure 2, which depicts an investor's optimal signal precision q_i as a function of the average private signal precision $\bar{q}$ for different degrees of benchmarking, as captured by the fraction of benchmarked investors in the economy, Γ. Due to strategic substituability (see, e.g., Grossman and Stiglitz (1980)), the investor's optimal signal precision is declining in the average private signal precision—irrespective of the degree of benchmarking (see also the explicit expression (A12)). Intuitively, a higher average private signal precision implies that more information is revealed through the public price signal, thereby reducing the investor's incentive to acquire private information himself.

However, most important for us is that the investor's optimal private signal precision shifts down as the degree of benchmarking increases—irrespective of the average private signal precision. This result is driven by *information-scale effects.* Intuitively, when making information choices in period 1, each investor take into account the *expected number of shares in his portfolio that are sensitive to private information.* In particular, one can re-write investor i's time-2 expected squared Sharpe ratio (divided by ρ) as:

$$\frac{z_i^2}{\rho} = h_i \, \frac{\hat{\mu}_{X,i} - PR_f}{\rho} \times (\hat{\mu}_{X,i} - PR_f) = \theta_i^{MV} \times (E_2[X] - PR_f)\,.$$

Hence, the benefit of private information in (11) is governed by the product of the mean-variance portfolio (the information-sensitive part of the portfolio) and the time-2 expected excess stock return. Taking into account market-clearing, the aggregate hedging demand reduces the expected number of shares in the information-sensitive part (θ_i^{MV}).[24] Because returns to information are increasing in the number of shares (i.e., one piece of information can be used for many shares), this implies a decline in the marginal value of private information for all investors. Alternatively, the aggregate hedging demand increases the stock price for all future states (see Theorem 2), thereby ambiguously reducing its expected excess return ($E_2[X] - PR_f$). This, in turn, reduces the potential rent from speculating with private information and, hence, the value of private information.

It is instructive to also study the case of CARA expected utility. The optimal stock demand and the resulting price function are identical to the case of CARA utility with a preference for the early resolution of uncertainty and are given by (6) and (7). However, at the information choice stage, an investor's best information choice differs. In particular, given arbitrary precision choices by the other investors, his optimal information choice is characterized by the following theorem:

Theorem 4. *Assume that investors have CARA expected utility. Conditional on the average private signal precision, $\bar{q}$, investor i chooses a signal of precision $q_i(\bar{q})$ such that:*

$$2\,\kappa'(q_i) = \frac{1}{\rho}\,\frac{1}{h_0 + q_i}.$$

Hence, the marginal benefit of private information is governed by the investor's posterior precision $h_0 + q_i$ and his risk-aversion ρ. Importantly, his optimal information choice q_i does *not* depend on the degree of benchmarking in the economy, $\bar{\gamma}$; implying that information-scale effects are absent. Graphically, this implies that while the investor's best information response is declining in the average private signal precision $\bar{q}$, the three lines in Figure 2 (for different fraction of benchmarked investors) would coincide.

[24] In particular, for each investor, the time-1 expectation of the number of shares in his portfolio that are sensitive to private information is lowered by $\bar{\gamma}$ relative to the absence of benchmarking.

We can now return to the case of mean-variance preferences. Recall that Theorem 3 takes the information choices of the other investors as given. However, in equilibrium, the information choice of each investor, q_i, affects aggregate average precision, $\bar{q}$, which, in turn, affects each investor's information choice. Therefore, the equilibrium value of the average private signal precision, $\bar{q}$, is determined by plugging q_i in (12) into its definition in (9):

Theorem 5. *The average private signal precision, $\bar{q}$, is the unique solution to:*

$$\bar{q} = \int_0^{\Gamma} q_i^{\mathcal{BI}}(\bar{q})\, di + \int_{\Gamma}^{1} q_i^{\mathcal{NI}}(\bar{q})\, di = \int_0^1 q_i(\bar{q})\, di. \tag{13}$$

If $\mu_Z - \bar{\gamma} > 0$, the average private signal precision, $\bar{q}$, is declining in the degree of benchmarking, $\bar{\gamma}$. Formally, $d\bar{q}/d\bar{\gamma} < 0$.

This theorem summarizes the key result of this section: An increase in the degree of benchmarking in the economy, $\bar{\gamma}$, leads to a decline in the precision of the private information of all investors (for the economically relevant case of $\mu_Z - \bar{\gamma} > 0$). Graphically, this result is illustrated in Figure 2, which shows that the investors' equilibrium precision choice, characterized by the intersection between the respective best information response function and the 45-degree line, declines in the fraction of benchmarked investors, Γ. Panel A of Figure 3 shows this even more explicitly by depicting the optimal precision of the investors' private signals, q_i, as a function of the fraction of benchmarked investors, Γ. As a result of the lower precision choices, equilibrium price informativeness, given by $\bar{q}^2/(\rho^2\, \sigma_Z^2)$, also declines in the degree of benchmarking. This is illustrated in Panel B of Figure 3.

REMARK 1. It is also instructive to consider the case of a single (small) benchmarked institutional investor with constant absolute risk-aversion and a preference for the early resolution of uncertainty. Because the marginal value of private information $A(\bar{\gamma}, \bar{q})$ does not depend on the strength of the investor's *individual* benchmarking concerns, γ_i, but only on the *aggregate* degree of benchmarking $\bar{\gamma}$, such an investor's information choice is actually not affected. Intuitively, the expected number of shares in his portfolio that are sensitive to private information does not decline in the absence of an aggregate hedging demand.

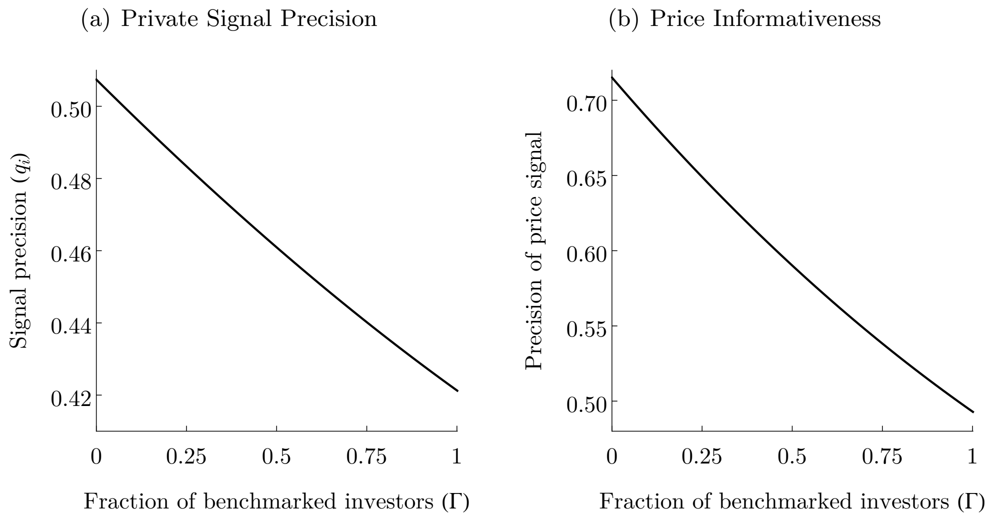

Figure 3: Equilibrium information choice. The figure shows the optimal private signal precision, q_i, in equilibrium (Panel A) and the precision of the public price signal ("price informativeness") $\bar{q}^2/(\rho^2\,\sigma_Z^2)$ (Panel B), as functions of the fraction of benchmarked investors in the economy, Γ. The graphs are based on the framework described in Section 2.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = 1.0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BM}$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

2.4 Unconditional Asset Prices and Return Moments

Benchmarking also affects the unconditional stock price and return moments. The following theorem summarizes a first set of results:

Theorem 6. *The total derivative of the unconditional stock price $S \equiv E_1[PR_f]$ and of the unconditional expected excess return $M \equiv E_1[X - PR_f]$ with respect to the degree of benchmarking, $\bar{\gamma}$, are given by:*

$$\frac{dS}{d\bar{\gamma}} = \frac{1}{\bar{h}}\,\rho + \frac{1}{\bar{h}^2}\,\rho\,(\mu_Z - \bar{\gamma})\,\frac{d\bar{h}}{d\bar{\gamma}}, \qquad \text{and} \qquad \frac{dM}{d\bar{\gamma}} = \frac{-\rho}{\bar{h}} + \frac{-\rho}{\bar{h}^2}\,(\mu_Z - \bar{\gamma})\,\frac{d\bar{h}}{d\bar{\gamma}}.$$

The stock price is affected directly through the excess demand and indirectly by the induced change in price informativeness. The direct effect (first component) leads to an increase in the price. The indirect effect (second component) is of the opposite sign (if $\mu_Z - \bar{\gamma} > 0$) and lowers the price. Intuitively, with lower price informativeness risk-averse investors command a price discount. In equilibrium, the total impact on the stock price depends on the relative importance of the two effects. We find that for realistic parameter

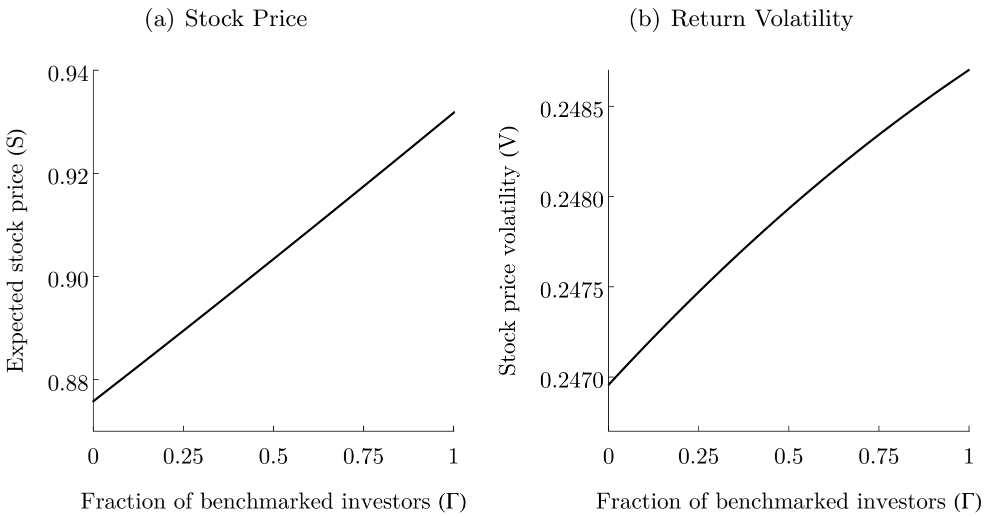

Figure 4: Equilibrium price and return volatility. The figure shows the unconditional expected equilibrium stock price S (Panel A) and the unconditional stock return volatility V (Panel B), as functions of the fraction of benchmarked investors in the economy, Γ. The graphs are based on the framework described in Section 2.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = 1.0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BM}$ and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

values the first effect always dominates; that is, the unconditional price increases, as illustrated in Panel A of Figure 4. The impact of benchmarking on the expected excess return follows accordingly but is of opposite sign. The direct (indirect) effect reduces (increases) the expected return because it increases (reduces) the stock price.

Finally, benchmarking also affects the unconditional stock return variance:

Theorem 7. *The total derivative of the unconditional return variance* $V^2 \equiv Var_1(X - PR_f)$ *with respect to the degree of benchmarking,* $\bar{\gamma}$*, is given by:*

$$\frac{dV^2}{d\bar{\gamma}} = -\frac{1}{\bar{h}^3}\frac{d\bar{h}}{d\bar{\gamma}}.$$

In particular, an increase in the degree of benchmarking, $\bar{\gamma}$, unambiguously leads to an increase in the unconditional return variance—due to the decline in price informativeness ($d\bar{h}/d\bar{\gamma} < 0$). This is illustrated in Panel B of Figure 4.

3 Benchmarking and Risk-Taking Effects

In this section, we discuss a second channel through which benchmarking affects information choice. For this purpose, we rely on a model with CRRA-preferences which, however, is considerably less tractable.[25] In particular, we demonstrate that benchmarking limits an investor's willingness to speculate. As a result, information aggregation is now also adversely affected. Hence, this "risk-taking effect" amplifies the decline in price informativeness. Moreover, benchmarked investors choose a lower precision of private information and, as a result, have lower expected portfolio returns than non-benchmarked investors.

3.1 Setup

We assume that investors have CRRA preferences. Their time-1 utility function $U_{1,i}^{\text{CRRA}}$ is given by:[26]

$$U_{1,i}^{\text{CRRA}} = E_1\left[E_2\left[\frac{C_i^{1-\rho}}{1-\rho}\right]\right] = E_1\left[\frac{C_i^{1-\rho}}{1-\rho}\right], \tag{14}$$

where ρ denotes the curvature parameter in utility and $C_i = W_i - \gamma_i\, W_{0,i}\,(X/P) - \kappa(q_i)$ denotes the investor's compensation, with $R_B \equiv (X/P)$ capturing the performance of the benchmark.[27,28]

For ease of exposition and numerical convenience, we assume that the payoff, $X \geq 0$, is binomially distributed, with equally likely realizations $X_H \equiv \mu_X + \sigma_X$ and $X_L \equiv \mu_X - \sigma_X$. Similarly, the investors' private signals, $Y_i \in \{Y_H, Y_L\}$, are binomially distributed.[29] Intuitively, a higher precision of the private signal, q_i, increases an investor's posterior precision by increasing the correlation between the payoff, X, and his private signal, Y_i;

[25] The equilibrium price function is nonlinear, and, hence, the model has to be solved numerically. Appendix B provides the technical details for our novel numerical solution approach.

[26] This is a special case of the general form in (2), with $u_1(x) = x$ and $u_2(x) = x^{1-\rho}/(1-\rho)$.

[27] Note, if a benchmarked investor puts all his wealth into the benchmark, his compensation is given by $C_i = (1-\gamma_i)\, W_{0,i}\, R_B - \kappa(q_i)$. Thus, we assume that $\gamma_i << 1$, such that buying the benchmark is always a feasible strategy that yields a strictly positive compensation and, hence, strictly positive marginal utility.

[28] In contrast to the CARA framework (in which one usually works with dollar returns as they lead to explicit expressions), we now directly work with percentage returns—as is done in practice.

[29] The results are robust to incorporating continuous distributions. In particular, the results are qualitatively unchanged if one relies on log-normal distributions for the payoff and the private signals, as discussed in Appendix C and illustrated in Figure A2 therein.

formally, $\mathbb{P}[X_o \,|\, Y_i = Y_o] = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{q_i}{q_i+4}}$, $o \in \{H, L\}$. To guarantee a positive supply of the stock and rule out negative compensation (for which CRRA preferences are undefined), we assume that the supply of the stock, Z, is log-normally distributed, with $\ln Z \sim \mathcal{N}(\mu_Z, \sigma_Z^2)$.

It is important to highlight that, in this setting, the strength of an individual investor's benchmarking concerns, γ_i, has an impact on his local risk-aversion:

Lemma 1. *The local curvature of the CRRA-utility function* (14) *with respect to terminal wealth, W_i; that is, the local coefficient of relative risk-aversion, $\hat{\rho}_i$, is given by:*

$$\hat{\rho}_i = \rho \, \frac{W_i}{W_i - \gamma_i \, W_{0,i} \, R_B - \kappa(q_i)}. \tag{15}$$

If $\gamma_i > 0$, it holds that $\hat{\rho}_i \geq \rho$ and $\partial \hat{\rho}_i / \partial \gamma_i \geq 0$, with strict inequalities for the case $R_B > 0$ (or, equivalently, $X > 0$).

Lemma 1 shows that the risk-aversion of a benchmarked investor, $\hat{\rho}_i$, exceeds the risk-aversion of a non-benchmarked investor (equal to ρ) and is increasing in the strength of his benchmarking concerns γ_i.[30] Intuitively, keeping $\gamma_i \, W_{0,i} \, R_B$ fixed, a given proportional movement in wealth W_i has a stronger impact on utility (14) if the wealth in excess of the benchmarking component, $W_i - \gamma_i \, W_{0,i} \, R_B$, is low.[31] Consequently, variations in the fraction of benchmarked investors, Γ, imply changes in the aggregate risk-bearing capacity in the economy; in particular, an increase in the fraction of benchmarked institutions leads to a *decline in the aggregate risk-bearing capacity.*

Note that, for all graphical illustrations, we again assume that the strength of the benchmarked investors' benchmarking concerns coincide: $\gamma_i = \gamma > 0, \forall i \in \mathcal{BI}$, while non-benchmarked investors have no relative performance concerns: $\gamma_i = 0$, $\forall i \in \mathcal{NI}$. As a result, all variation in the degree of benchmarking in the economy is driven by changes in the fraction of benchmarked investors, Γ.

[30] Risk-aversion is also increasing in the benchmark portfolio's return; formally, $\partial \hat{\rho}_i / \partial R_B > 0$.

[31] In that regard, benchmarking concerns are akin to a subsistence level or background risk.

3.2 Portfolio Choice

At time $t = 2$, an investor must choose his optimal portfolio in order to maximize (3), conditional on his posterior beliefs, described by $\mathcal{F}_i$, and taking the price P as given. For illustrative purposes, one can derive the following approximate solution for the optimal fraction of wealth invested into the stock, $\phi_i \equiv (\theta_i P)/W_{0,i}$:[32]

$$\phi_i \approx \frac{E_2[r^e \,|\, \mathcal{F}_i]}{\frac{\rho}{1-\gamma_i} Var_2[r^e \,|\, \mathcal{F}_i]} + \gamma_i \equiv \phi_i^{MV} + \gamma_i, \tag{16}$$

where r^e denotes the stock's excess return, $r^e \equiv X/P - R_f$.

The optimal demand for the stock has the same two components as in the case of constant absolute risk-aversion. First, the mean-variance portfolio, ϕ_i^{MV}. But, note that this is now the mean-variance portfolio of an investor with *local risk-aversion* $\rho/(1-\gamma_i)$.[33] As a result, it is declining in the strength of an investor's benchmarking concerns, γ_i. Second, the hedging portfolio, γ_i, which is increasing in an investor's benchmarking concerns, but is information-insensitive. Naturally, for non-benchmarked investors ($\gamma_i = 0$), the demand reduces to the standard mean-variance portfolio of an investor with risk-aversion ρ.

Figure 5 illustrates how the optimal stock demand of a benchmarked investor varies with the strength of his benchmarking concerns, γ_i. In particular, Panel A shows that the overall portfolio share of the stock, ϕ_i, is increasing in an investor's benchmarking concerns γ_i—comparable to the case of constant absolute risk-aversion. But, due to the decline in the mean-variance portfolio, ϕ_i^{MV}, the overall increase is substantially smaller.

Panel B further illustrates that effect. It shows an investor's optimal stock demand, conditional on a specific signal realization $Y_i \in \{Y_H, Y_L\}$. As expected, the investor, in general, over-weights the stock following a high signal, Y_H, and vice versa for a low signal,

[32] The approximation relies on approximating (normalized) compensation, $\tilde{C}_i$ (see (B1) in Appendix B), using a log-normally distributed variable. It is valid if $\left(1/(1-\gamma_i)\right)\left(W_{0,i}\left(R_f + (\phi_i - \gamma_i)E_1[r^e] - \kappa(q_i)\right)\right.$ is around one and, e.g., very accurate if the risk-free rate is below 5% and the expected excess return is below 10%. See also van Nieuwerburgh and Veldkamp (2010).

[33] This is consistent with Lemma 1. In particular, a benchmarked investor's risk-aversion $\hat{\rho}_i$ can be rewritten as $\rho \,/ \left(1 - \gamma_i \frac{W_{0,i} R_B - \kappa(q_i)/\gamma_i}{W_i}\right)$. In a one-stock economy and ignoring the (small) information cost, $\kappa(q_i)$, the last part is given by $(1 + r^e)/(R_f + \phi r^e)$, which, for realistic values of R_f (around 1.0) and ϕ (around 1.0), is close to 1, such that $\hat{\rho}_i$ reduces to $\rho/(1-\gamma_i)$.

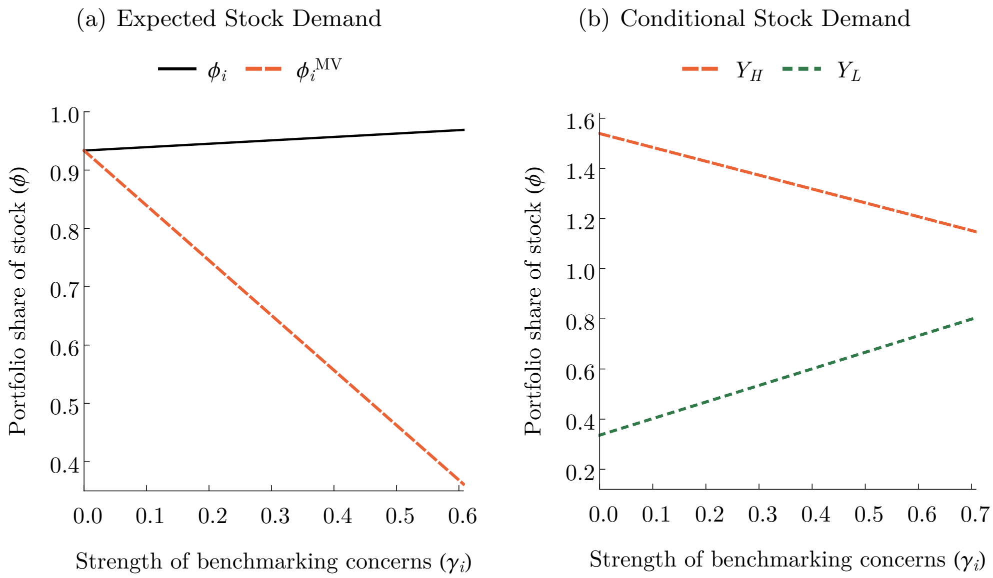

Figure 5: Stock demand. The figure illustrates a benchmarked investor's portfolio choice for various levels of the strength of his benchmark concerns, γ_i and conditional on a given signal precision q_i. Panel A shows the expected stock demand and the mean-variance portfolio component. Panel B depicts the stock demand conditional on a specific signal realization $Y_i \in \{Y_H, Y_L\}$. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $q_i = 0.1667$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

Y_L. Most importantly, however, the "spread" between a benchmarked investor's stock demand following a positive and a negative signal narrows as his benchmarking concerns, γ_i, strengthen. Hence, in the presence of benchmarking, the investor's willingness to speculate declines (due to his higher local risk-aversion). This result is in clear contrast to the case of constant absolute risk-aversion (with or without a preference for the early resolution of uncertainty), for which the mean-variance portfolio component is independent of the strength of an investor's benchmarking concerns.

3.3 Information Choice

To build the basic intuition for the incremental impact of the risk-taking effect on information choice in the clearest possible way, consider first the case of a single (small) benchmarked investor. Hence, benchmarking does not affect the equilibrium stock price or the effective supply. Recall from Remark 1, that, in this case, the strength of the investor's individual benchmarking concerns, γ_i, does not affect his information choice with mean-variance preferences.

This result does not hold with CRRA preferences. In particular, Panel A of Figure 6 depicts the optimal signal precision q_i of a single (small) benchmarked investor as a function of the average private signal precision $\bar{q}$—for different levels of the strength of his benchmarking concerns, γ_i. As expected, the investor's optimal signal precision is declining in the average private signal precision. Most importantly, the optimal signal precision is now also declining in the strength of the investor's *individual* benchmarking concerns, γ_i. Intuitively, the investor anticipates that his time-2 portfolio choice will be less sensitive to the realization of his private signal because he trades less aggressively, and, consequently, the value of private information declines. Panel B shows this even more explicitly by depicting the optimal precision of the investor's private signal, q_i, as function of the strength of his benchmarking concerns, γ_i.

Note that, because information-scale effects are also present with CRRA preferences, varying the degree of benchmarking in the economy $\bar{\gamma}$ (e.g., the fraction of benchmarked investors), would also lead to a decline in the optimal signal precision—similar to the case of mean-variance preferences (as is illustrated in Figure 2). However, the key new result is that the investor's individual benchmarking concerns γ_i now also play a role.

The lower value of information for benchmarked investors implies that, in equilibrium, benchmarked institutions collect less information than non-benchmarked institutions—in stark contrast to the earlier results (in which both groups of market participants chose the same signal precision). This is illustrated in Panel A of Figure 7.

The figure also shows the novel result that both groups of investors choose a higher precision for their private signals as the share of benchmarked investors, Γ, increases. To understand this result, recall that an increase in the fraction of benchmarked investors leads to a decline in the aggregate risk-bearing capacity. Consequently, prices reveal less information which, in turn, increases the marginal benefit of private information and, hence, the incentives of all investors to choose a more precise signal.

However, as shown in Panel B of Figure 7, price informativeness—the amount of private information aggregated and revealed by the stock price—is declining in the fraction of benchmarked investors. This result is driven by two distinct economic forces: information

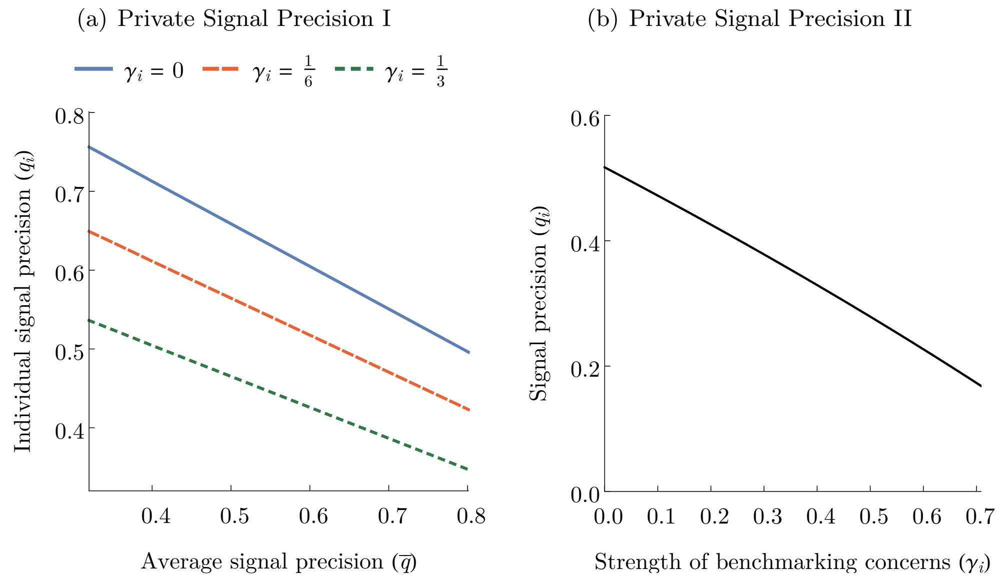

Figure 6: Information demand. The figure depicts the optimal signal precision q_i of a single (small) benchmarked investor. Panel A shows the optimal signal precision as a function of the average private signal precision $\bar{q}$—for different levels of the strength of his benchmark concerns, γ_i. Panel B shows the optimal signal precision as function of the strength of the investor's benchmark concerns, γ_i. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

acquisition and information aggregation. First, an increase in the share of benchmarked investors implies that better-informed non-benchmarked investors are replaced by less-informed benchmarked investors (as illustrated in Panel A). Second, it implies a shift toward a group of investors that trades less aggressively based on available private information—because of their lower risk appetite. Consequently, less information can be aggregated into the price. Both effects imply a decline in price informativeness.

To (quantitatively) disentangle this effect from the information-scale effect, Panel B of Figure 7 also shows how price informativeness varies with the fraction of benchmarked investors, if one would keep the aggregate risk-bearing capacity fixed.[34] While the majority of the decline is due to the reduction in aggregate risk-bearing capacity, the impact of the information-scale effect is non-negligible (about 25% of the decline for the parameters used for the illustration).

[34]For that purpose, we reduce the curvature parameter of benchmarked investors' utility, such that their risk-aversion, after accounting for their benchmarking concerns, equals that of non-benchmarked investors. In particular, we choose the curvature parameter such that a single benchmarked investor's mean-variance portfolio component matches that of a non-benchmarked investor.

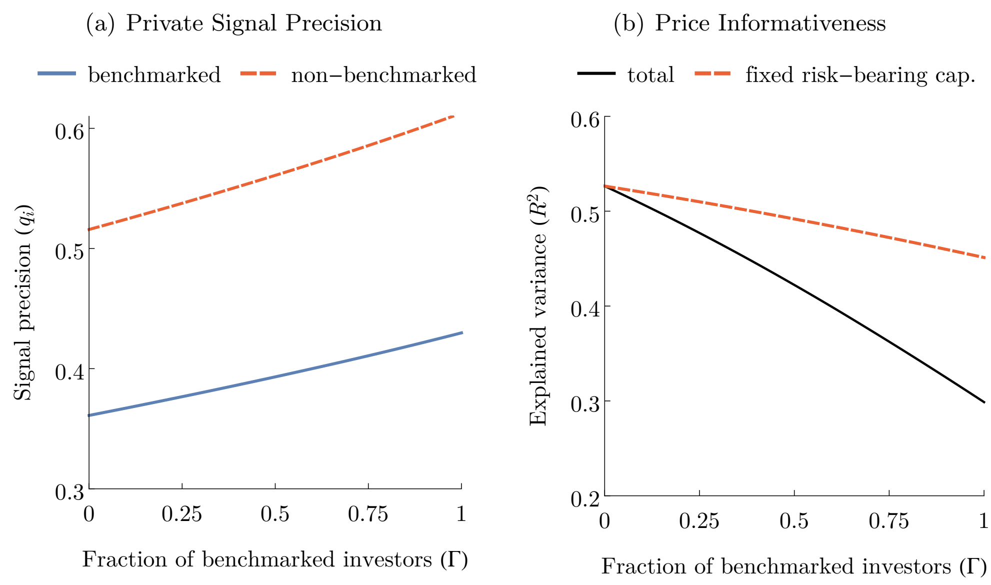

Figure 7: Equilibrium information choice. The figure illustrates equilibrium signal precisions, as functions of the fraction of benchmarked investors in the economy, Γ. Panel A shows the optimal private signal precision of the two groups of investors and Panel B depicts the precision of the public price signal ("price informativeness"). Precision is measured as R^2, that is, the fraction of the variance of the payoff X that is explained by the investors' private information and the stock price, respectively. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BI}$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

3.4 Unconditional Asset Prices and Return Moments

Similar to the case of mean-variance preferences, the unconditional stock price is affected through two effects which are in conflict, such that the net effect depends on their relative importance. On the one hand, the aggregate hedging demand pushes up the price. On the other hand, the decline in price informativeness increases posterior uncertainty, and, hence, risk-averse investors command a lower price.

Panel A of Figure 8 shows that, due to the amplification of the decline in price informativeness, the *stock price can decline in the fraction of benchmarked investors.* This is the case if information acquisition costs are low (ω_{low}) and, hence, price informativeness declines from a high initial level. This result is distinctly different from the setting with mean-variance preferences in which—for realistic parameters values—the stock price always increases in the fraction of benchmarked investors. As one increases information acquisition costs, the stock price first becomes essentially flat (ω_{high}) and in the limit approaches the

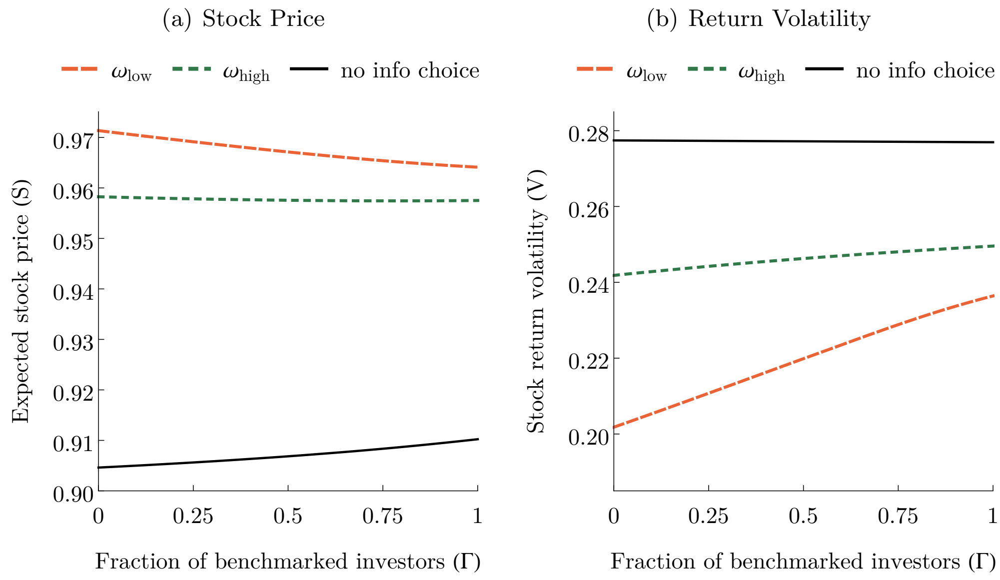

Figure 8: Equilibrium price and return volatility. The figure shows the unconditional expected equilibrium stock price (Panel A) and the unconditional stock return volatility (Panel B), as functions of the fraction of benchmarked investors in the economy, Γ. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BI}$ and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega_{low} = 0.015$, and $\omega_{high} = 0.045$.

case without information choice in which the price is always increasing in the fraction of benchmarked investors. The implications for the expected excess return are simply of the opposite sign.

Similar to the case of mean-variance preferences, the stock's return volatility is unambiguously increasing in the share of benchmarked investors because of the higher posterior uncertainty. This is illustrated in Panel B of Figure 8. The effect is quantitatively stronger for lower information acquisition costs and unique to our setting with endogenous information choice (at least for static frameworks).

3.5 Portfolio Returns

In practice, institutional investors typically invest on behalf of their clients. Thus, the differences in the institutional investors' portfolio and information choices (resulting from benchmarking) will also affect the clients' expected returns.

As shown in Panel A of Figure 9, the difference between the expected portfolio return of non-benchmarked and benchmarked institutional investors is always positive. Hence, non-benchmarked investors generate higher expected returns than their benchmarked peers. Moreover, as the fraction of benchmarked investors increases, this "return gap" widens.

To understand this effect, recall that benchmarked investors always choose a lower precision of private information than non-benchmarked investors (see Panel A of Figure 7). This has two effects for expected portfolio returns. First, non-benchmarked investors hold, on average, more of the stock, because their superior information renders the investment less risky. Due to the stock's positive risk premium, this increases their expected portfolio return. Second, conditional on a signal realization, non-benchmarked investors' trades are more profitable. Both effects unambiguously lead to a higher expected portfolio return for non-benchmarked investors. As the fraction of benchmarked investors increases, price informativeness declines and less information is revealed through the public stock price. Thus, the importance of private information rises. Consequently, the "information gap" between investors' information sets $\mathcal{F}_i$ widens (Panel B of Figure 9) and so does the "return gap." Note that these effects are not present for mean-variance preferences because in this case the precision of private information of benchmarked and non-benchmarked institutional investors coincides.

3.6 Robustness

The results are robust to changes in the parameter values. Intuitively, the increase in the benchmarked investors' risk-aversion, as discussed in Lemma 1, does not rely on specific parameter values. Hence, information aggregation is always adversely affected. Moreover, their higher local risk-aversion implies that benchmarked investors choose less precise private information. Consequently, the difference in the expected portfolio returns between benchmarked and non-benchmarked investors is positive and increases in the fraction of benchmarked investors.

The only quantities for which the impact of benchmarking depends on the choice of parameter values are, as discussed above, the unconditional expected stock price and excess

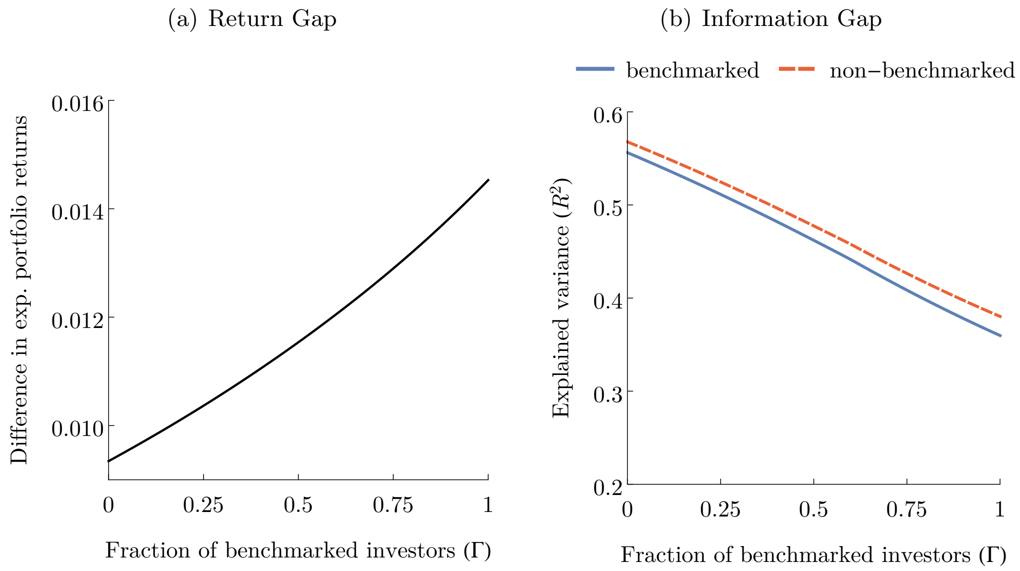

Figure 9: Return and information gap. The figure illustrates the return and information gap between the two groups of institutional investors, as functions of the fraction of benchmarked investors in the economy, Γ. Panel A shows the difference between the expected portfolio return of non-benchmarked and benchmarked institutions. Panel B shows the precision of investors' information sets, $\mathcal{F}_i = \{Y_i, P\}$, measured as a fraction of the variance of the payoff X that they can explain. The graphs are based on the CRRA framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BI}$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

return. In both cases, there are two economic forces that are in conflict, with the net effect depending on their magnitudes.

4 Extensions

This section introduces two extensions of our basic economic framework. First, an extension to multiple stocks and, second, an extension to asymmetric benchmarking concerns. In both cases, we rely on CRRA preferences, capturing information-scale and risk-taking effects.

4.1 Multiple Stocks

Our objective in this section is to study how the results generalize in an economy with multiple stocks. In particular, we consider an economy with two symmetric stocks, $k \in \{1, 2\}$, in which each investor has to decide simultaneously on the signal precision for the two stocks, $q_{i,k}$. Similar to the one-stock setup described in Section 3.1, the stocks' payoffs,

X_k, and the private signals, $Y_{i,k}$, are assumed to be binomially distributed, and the supply Z_k is assumed to be log-normal. In addition, we assume that payoffs, signals, and supplies are independent across assets.[35] The first stock (the "index stock") also serves as the benchmark such that the compensation of the benchmarked investors is declining in its (gross) return: $\gamma_{i,1} > 0, \forall i \in \mathcal{BI}$. In contrast, the second stock (the "non-index stock") is not part of the benchmark: $\gamma_{i,2} = 0, \forall i$.

Panel A of Figure 10 illustrates that price informativeness is declining for both stocks in the fraction of benchmarked investors, Γ. However, the decline in price informativeness is more pronounced for the index stock. Intuitively, the increase in the benchmarked investors' risk-aversion limits their speculative activities in the non-index stock as well.[36] However, because of the absence of an information-insensitive aggregate hedging demand, information-scale effects are absent. Hence, the expected number of shares of the non-index stock in investors' portfolios that are sensitive to private information does not decline. Consequently, a piece of information can be applied to more shares, and, hence, the value of private information and price informativeness are higher.

Panel B shows that the price of the index stock is always higher than that of the non-index stock, with the price gap widening as the fraction of benchmarked investors increases. Intuitively, the aggregate hedging demand for the index stock increases its price relative to the non-index stock. At the same time, the stronger decline in price informativeness for the index stock implies more of a discount. In equilibrium, the first effect is quantitatively stronger, explaining the observed price pattern. The results for the stocks' expected returns follow accordingly. For both stocks, the unconditional return volatility is increasing because price informativeness declines. However, due to the more pronounced decline in the index stock's price informativeness, the increase in its return volatility is stronger as well. Finally, non-benchmarked investors' private information in both assets is more precise, so that the results regarding their superior expected portfolio return carry over.

[35] We focus on the case of stocks with symmetric distributions of independent fundamentals and noise, so that all differences between the two stocks arise exclusively from benchmarking.

[36] Technically, conditional on a benchmarked investor's posterior beliefs, his optimal demand for the index stock is characterized by (16), with $\gamma_{i,1} > 0, \forall i \in \mathcal{BI}$. Moreover, his demand for the non-index stock is also described by (16), but with $\gamma_{i,2} = 0$.

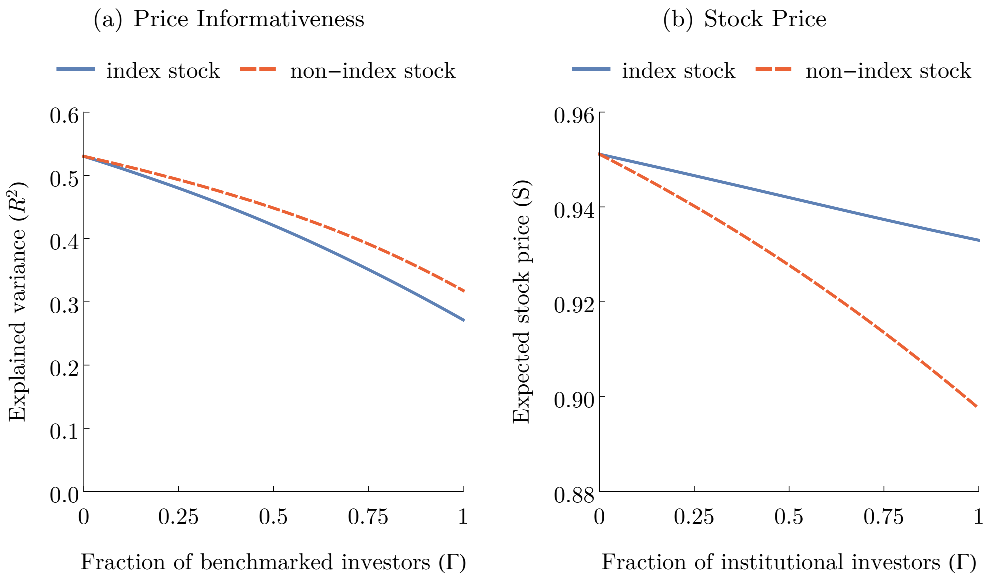

Figure 10: Multiple stocks. The figure shows price informativeness and the unconditional expected stock price of the index and non-index stock, as functions of the fraction of benchmarked investors in the economy, Γ. Panel A depicts the precision of the public price signal ("price informativeness"), measured as the fraction of the variance (R^2) of the payoff X_k that is explained by the corresponding stock price. Panel B shows the unconditional expected equilibrium stock price. The graphs are based on the symmetric two-stock CRRA framework described in Section 4.1, with the following parameter values: $\mu_{X,k} = 1.05$, $\sigma^2_{X,k} = 0.25$, $\mu_{Z,k} = \ln(1) = 0$, $\sigma^2_{Z,k} = 0.2$ for $k \in \{1, 2\}$, $\rho = 3$, $\gamma_{i,1} = \gamma = 1/3\,\forall i \in \mathcal{BM}$, $\gamma_{i,2} = 0\,\forall i$, and an information cost function $\kappa(q_{i,k}) = \omega\, q^2_{i,k}$, $k \in \{1, 2\}$, with $\omega = 0.015$.

4.2 Asymmetric Benchmarking Concerns

Second, we consider an extension to asymmetric benchmarking concerns. In particular, while U.S. mutual funds are limited to use linear (Fulcrum) performance fees, asymmetric performance fees for mutual funds can be used outside the United States and are also much more standard in other asset classes (such as hedge funds or private equity). Moreover, implicit benchmarking concerns at the fund manager's level or arising from the flow-performance relation are typically also rather asymmetric.[37]

The easiest way to incorporate benchmarking concerns that are nonlinear in the benchmark's return is to model a benchmarked investor's compensation, C_i, as:

$$C_i(W_i, R_B) = W_i - W_{0,i}\, R_B\, \left(\gamma_i^+\, \mathbb{1}_{R_B>1} + \gamma_i^-\, \mathbb{1}_{R_B\leq 1}\right) - \kappa(q_i),$$

[37] See, e.g., Ibert, Kaniel, van Nieuwerburgh, and Vestman (2018) for empirical evidence on Swedish mutual fund managers and Sirri and Tufano (1998), Chevalier and Ellison (1997) and Huang, Wei, and Yan (2007) for evidence on an asymmetric flow-performance relation.

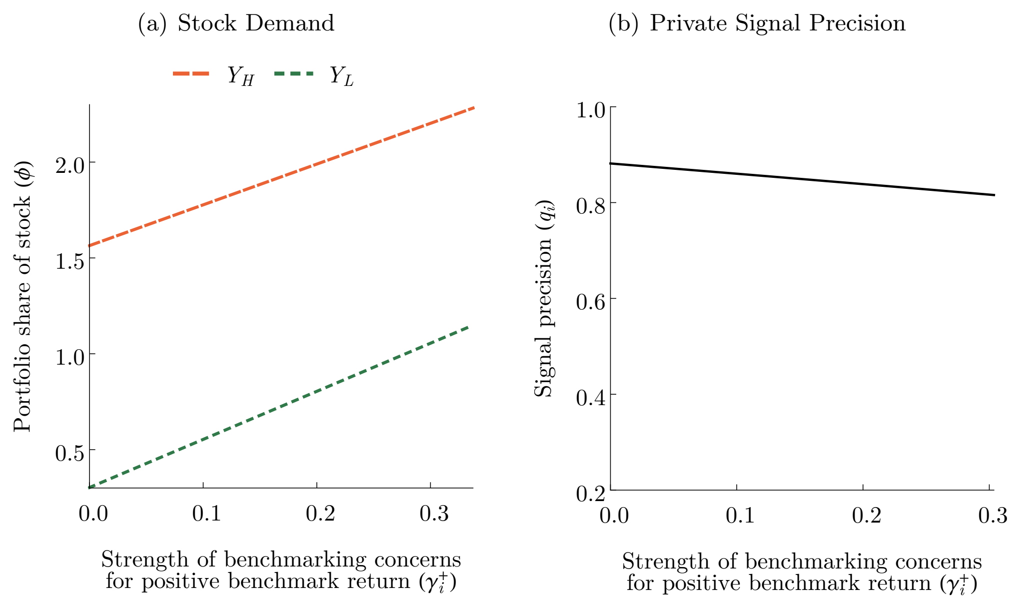

Figure 11: Asymmetric benchmarking concerns. The figure illustrates a single (small) benchmarked investor's stock and information demand, as functions of the strength of his benchmarking concerns for positive benchmark returns γ_i^+. Panel A shows the stock demand conditional on a specific signal realization $Y_i \in \{Y_H, Y_L\}$, with exogenous signal precision $q_i = 0.1667$. Panel B depicts the optimal, endogenous signal precision q_i. The graphs are based on the framework described in Section 4.2, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i^- = 0$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

where $\mathbb{1}_\cdot$ denotes the indicator function. In particular, with this specification, the sensitivity of the investor's compensation with respect to positive (gross) benchmark returns, γ_i^+, can differ from the sensitivity with respect to negative (gross) benchmark returns, γ_i^-.[38] Otherwise, the setup is similar to the one described in Section 3.1.

As a full analysis of the equilibrium with asymmetric benchmarking concerns is outside the scope of this paper, we concentrate on the case of a single (small) benchmarked investor with asymmetric benchmarking concerns; in which case benchmarking does not affect the equilibrium stock price. In particular, consider the case of varying the strength of the investor's benchmarking concerns for positive benchmark returns, γ_i^+, while setting the strength of his benchmarking concerns for negative benchmark returns, γ_i^-, to zero.[39]

Panel A of Figure 11 shows the investor's stock demand conditional on a specific signal realization $Y_i \in \{Y_H, Y_L\}$. Interestingly, the investor's willingness to speculate, measured

[38] Technically, this introduces at kink at $R_B = 1$, which further complicates the solution.

[39] The results are qualitatively unchanged if one varies the strength of the investor's benchmarking concerns for negative benchmark returns, γ_i^-, while setting the strength of his benchmarking concerns for positive benchmark returns, γ_i^+, to zero.

by the “spread” between his stock demand following a high and a low signal, is declining substantially less than in the case of symmetric benchmarking concerns (see Panel B of Figure 5). On the one hand, when the investor observes a signal indicating a low payoff and, hence, a negative benchmark return, the aggressiveness with which he trades is essentially unaffected because $\gamma_i^- = 0$. On the other hand, when the investor observes a signal indicating a high payoff and, hence, a positive benchmark return, the aggressiveness with which he trades is only marginally affected relative to the no-benchmarking case (although $\gamma_i^+ > 0$). Intuitively, because the investor over-weights the benchmark following a high signal (i.e., the stock's portfolio share is above 1), his expected final wealth, W_i, relative to the benchmark component $\gamma_i^+ R_B$, is high, limiting the increase in local risk-aversion (15).

Consequently, the sensitivity of the investor's portfolio with respect to private information declines only marginally as the strength of the investor's benchmarking concerns for positive benchmark returns, γ_i^+, increases. Hence, as shown in Panel B of Figure 11, the under-provision of private information seems to be considerably weaker than in the case of symmetric benchmarking concerns. That is, the investor chooses a substantially higher signal precision than in the linear case (see Panel B of Figure (6))—although the optimal signal precision is still declining in the strength of his benchmarking concerns.

5 Key Predictions and Conclusion

Relative performance concerns play a key role in the decisions of institutional investors who are in the business of acquiring information and using that information for portfolio management. In this paper, we develop an economic framework that explicitly accounts for benchmarking *and* investors' joint portfolio and information choice.

We identify two distinct economic channels through which benchmarking affects informational efficiency. Key to both economic mechanisms is the *interaction* between investors' portfolio and information choices. First, the information-insensitive aggregate hedging demand of benchmarked investors reduces, in equilibrium, the expected number of shares in investors' portfolios that are sensitive to private information. This information-scale effect reduces the value of private information but does not affect information aggregation. Second, an investor's individual benchmarking concerns limit his willingness to speculate, so that the sensitivity of his portfolio with respect to private information declines. This risk-taking effect leads to a decline in the value of private information but also adversely affects information aggregation. Notably, incorporating risk-taking effects leads to qualitatively different implications. For example, benchmarked and non-benchmarked investors differ in their information choice and, hence, in their expected portfolio returns. Moreover, the equilibrium stock price can decline in the fraction of benchmarked investors.

The model generates a rich set of predictions, which are empirically refutable. For example, similar to our results, the literature on "index effects" has found a higher price and a lower Sharpe ratio for index stocks relative to non-index stocks. While these results are not unique to our model (see, e.g., Cuoco and Kaniel (2011) and Basak and Pavlova (2013)), the joint analysis of portfolio and information choices also leads to novel and unique predictions. In particular, our model predicts a lower price informativeness for index stocks. In line with this prediction, Israeli, Lee, and Sridharan (2017) document that an increase in ETF ownership is associated with less-informative security prices. Also, our model makes unique predictions about the expected portfolio returns of benchmarked and non-benchmarked investors that would not arise in models with symmetric information and can be confronted with the data.

In this paper, the benchmarking concerns of the institutional investors are exogenous. However, in the presence of agency frictions, they might be an endogenous outcome; for example, the incentives between institutional investors and their clients (see, e.g., Buffa, Vayanos, and Woolley (2017) or Sotes-Paladino and Zapatero (2016)) as well as between fund managers within an asset management firm (see, e.g., van Binsbergen, Brandt, and Koijen (2008)) might be better aligned due to benchmarking. In this regard, our paper highlights a novel tension between benchmarking as a tool to align incentives and its adverse effects on managers' information acquisition and, hence, their portfolio returns.

Consequently, a logical extension of our work would be to study optimal asset management contracts in a setting with agency frictions *and* information acquisition.[40] In particular, allowing for nonlinear contracts might lead to many new insights regarding the optimal compensation in the asset-management industry. Also, extensions of our framework can be used to study the optimal size of benchmarked investors in the economy (Pástor and Stambaugh (2012)) or the implications of passive investing.

[40] Intuitively, because monitoring information acquisition is difficult, fund managers have an incentive to lie, which, naturally, creates an agency conflict.

Appendix

A Proofs for Section 2

Proof of Theorem 1

Plugging wealth W_i into an investor's compensation $C_i = W_i - \gamma_i\,(X - PR_f) - \kappa(q_i)$ and substituting into the time-2 objective function $E_2\,[C_i] - \rho/2\,Var_2\,(C_i)$ yields:

$$
\begin{aligned}
&E_2\,[W_{0,i}R_f + (\theta_i - \gamma_i)\,(X - PR_f) - \kappa(q_i)] - \frac{\rho}{2} Var_2\big((\theta_i - \gamma_i)\,(X - PR_f)\big) \\
&= W_{0,i}R_f - \kappa(q_i) + (\theta_i - \gamma_i)\,(\hat{\mu}_{X,i} - PR_f) - \frac{\rho}{2}\,(\theta_i - \gamma_i)^2\,\frac{1}{h_i}.
\end{aligned}
$$

Hence, the first-order condition with respect to θ_i is given by:

$$
(\hat{\mu}_{X,i} - P\,R_f) - \rho\,(\theta_i - \gamma_i)\,\frac{1}{h_i} \triangleq 0,
$$

which, after re-arranging, yields Theorem 1.

Proof of Theorem 2 and Corollary 1

We conjecture (and later verify) that the equilibrium stock price is a linear function of the unobservable state variables, that is, the payoff and the aggregate stock supply:

$$
PR_f = a + b\,X - d\,Z. \tag{A1}
$$

As a result, conditional on a specific signal Y_i and the realized price PR_f, investor i's posterior precision and mean are given by:

$$
h_i = \underbrace{\frac{1}{\sigma_X^2} + \frac{b^2}{d^2\,\sigma_Z^2}}_{\equiv h_0} + q_i = \frac{\left(1 + q_i\,\sigma_X^2\right)\,d^2\,\sigma_Z^2 + b^2\,\sigma_X^2}{\sigma_X^2\,d^2\,\sigma_Z^2}, \tag{A2}
$$

$$
\begin{aligned}
\hat{\mu}_{X,i} &= \mu_X + \frac{1}{h_i}\,q_i\,\left(Y_i - \mu_X\right) + \frac{1}{h_i}\,\frac{b^2}{d^2\,\sigma_Z^2}\left(\frac{P\,R_f - a + d\,\mu_Z}{b} - \mu_X\right) \qquad \text{(A3)} \\
&= \frac{d^2\,\sigma_Z^2\,\left(q_i\,\sigma_X^2\,Y_i + \mu_X\right) + b\left(PR_f - a + d\,\mu_Z\right)\sigma_X^2}{(1 + q_i\,\sigma_X^2)\,d^2\,\sigma_Z^2 + b^2\,\sigma_X^2},
\end{aligned}
$$

where we substituted the posterior precision h_i with (A2) in the last step.

Intuitively, investors have three pieces of information that they aggregate to form their expectation of the asset's payoff: their prior beliefs, their private signals and the public stock price. The posterior mean is simply the weighted average of the three signals' realizations, while the posterior precision is the weighted average of the signals' precisions.

Plugging the posterior mean and precision into the optimal stock demand in (6) yields:

$$\theta_i = \frac{1}{\rho}\left(\frac{\mu_x}{\sigma_X^2} + q_i\, Y_i + \frac{b\,(-a + d\mu_Z)}{d^2\sigma_Z^2} - P\,R_f\left(\frac{1}{\sigma_X^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1)\right)\right) + \gamma_i. \tag{A4}$$

Market clearing requires that aggregate demand, that is, demand (A4) integrated over all investors, equals (random) supply:

$$\int_0^{\Gamma}\left[\frac{1}{\rho}\left(\frac{\mu_x}{\sigma_X^2} + q_i\, Y_i + \frac{b\,(-a + d\mu_Z)}{d^2\sigma_Z^2} - P\,R_f\left(\frac{1}{\sigma_X^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1)\right)\right) + \gamma_i\right] di$$
$$+ \int_{\Gamma}^{1}\left[\frac{1}{\rho}\left(\frac{\mu_x}{\sigma_X^2} + q_i\, Y_i + \frac{b\,(-a + d\mu_Z)}{d^2\sigma_Z^2} - P\,R_f\left(\frac{1}{\sigma_X^2} + q_i + \frac{b}{d^2\sigma_Z^2}(b-1)\right)\right)\right] di \triangleq Z.$$

Substituting the private signals by $Y_i = X + \varepsilon_i$ and using that investors are, on average, unbiased ($\int_0^{\Lambda} \varepsilon_i = 0$, $\int_{\Lambda}^{1} \varepsilon_i = 0$), this can be simplified to:

$$\frac{1}{\rho}\left(\frac{\mu_x}{\sigma_X^2} + \bar{q}\, X + \frac{b\,(-a + d\mu_Z)}{d^2\sigma_Z^2} - P\,R_f\left(\frac{1}{\sigma_X^2} + \bar{q} + \frac{b\,(b-1)}{d^2\sigma_Z^2}\right)\right) + \bar{\gamma} \triangleq Z, \tag{A5}$$

with the precision of private information of the average agent, $\bar{q}$, and the degree of benchmarking in the economy, $\bar{\gamma}$, being defined in (9).

Solving the market-clearing condition (A5) for the price PR_f yields:

$$PR_f = \frac{d^2\sigma_Z^2}{\bar{h}\, d^2\sigma_Z^2 - b}\left(\frac{\mu_x}{\sigma_X^2} + \rho\,\bar{\gamma} + \frac{b\,(-a + d\mu_Z)}{d^2\sigma_Z^2}\right) + \frac{\bar{q}\, d^2\sigma_Z^2}{\bar{h}\, d^2\sigma_Z^2 - b}\, X - \frac{\rho\, d^2\sigma_Z^2}{\bar{h}\, d^2\sigma_Z^2 - b}\, Z,$$

with $\bar{h}$ being defined in (8). Moreover, it implies a signal-to-noise ratio, defined as the ratio of the price sensitivities with respect to the payoff and the noisy supply b/d, of $-\bar{q}/\rho$.

Finally, matching the coefficients of this price function to the ones of the conjecture (A1) and solving the resulting equation system for a, b and d yields the equilibrium price function (7) Finally, plugging these coefficients into h_0 defined in (A2) yields its expression in (8) and Corollary 1.

Proof of Theorem 3

Plugging the optimal portfolio θ_i in (6) into compensation C_i and computing its expectation and variance, yields:

$$E_2[C_i] = W_0\, R_f + h_i\, \frac{(\hat{\mu}_{X,i} - P\, R_f)^2}{\rho} - \kappa(q_i) = R_f\, \left(W_0 - \frac{\kappa(q_i)}{R_f}\right) + \frac{1}{\rho}\, z_i^2, \tag{A6}$$

$$Var_2(C_i) = h_i^2\, \left(\frac{\hat{\mu}_{X,i} - P\, R_f}{\rho}\right)^2\, \frac{1}{h_i} = h_i\, \frac{(\hat{\mu}_{X,i} - P\, R_f)^2}{\rho^2} = \frac{1}{\rho^2}\, z_i^2, \tag{A7}$$

with z_i being defined as $z_i \equiv \sqrt{h_i}\, (\hat{\mu}_{X,i} - P\, R_f)$. Substituting the expectation and variance into the investor's utility function (5) gives the investor's time-1 objective function (11).

To ease the computations of $E_1[z_i^2]$, introduce $u_i = \sqrt{h_i} z_i = h_i(\hat{\mu}_{X,i} - P\, R_f)$. Moreover, note that viewed from period 1 both—$\hat{\mu}_{X,i}$ and PR_f—are random variables. In particular, substituting Y_i by $X + \varepsilon_i$ and $\frac{P\, R_f - a + d\, \mu_Z}{b}$ by $X - \frac{\rho}{\bar{q}}\, (Z - \mu_Z)$ in the expression for the posterior mean (A3) yields:

$$\hat{\mu}_{X,i} = \frac{1}{h_i}\, \left(\left(\frac{\mu_X}{\sigma_X^2} + \frac{\mu_Z\, \bar{q}}{\rho\, \sigma_Z^2}\right) + q_i\, \varepsilon_i + \left(q_i + \frac{\bar{q}^2}{\rho^2\, \sigma_Z^2}\right) X - \frac{\bar{q}}{\rho\, \sigma_Z^2}\, Z \right). \tag{A8}$$

Consequently, after substituting (A8) for $\hat{\mu}_{X,i}$ and (7) for PR_f in u_i, one gets:

$$u_i = \left(\frac{\mu_X}{\sigma_X^2} + \frac{\bar{q}\, \mu_Z}{\rho\, \sigma_Z^2}\right) \left(1 - \frac{h_i}{\bar{h}}\right) - \frac{h_i}{\bar{h}}\, \rho\, \bar{\gamma} + q_i\, \varepsilon_i + \left(\frac{1}{\sigma_X^2}\, \left(\frac{h_i}{\bar{h}} - 1\right)\right) X$$
$$+ \frac{\rho}{\bar{q}}\, \left(h_i - \frac{\bar{q}^2}{\rho^2\, \sigma_Z^2} - \frac{h_i}{\bar{h}\, \sigma_X^2}\right) Z.$$

Integrating over the distributions of P and Y_i, the time-1 expectation and variance are:

$$E_1[u_i] = \frac{h_i\, \rho}{\bar{h}}\, (\mu_Z - \bar{\gamma})\, ,$$

$$Var_1(u_i) = \frac{h_i^2}{\bar{h}^2}\, \left(\frac{1}{\sigma_X^2} + \frac{\rho^2}{\bar{q}^2}\, \sigma_Z^2\, \left(\bar{h} - \frac{1}{\sigma_X^2}\right)^2\right) - h_i = \frac{h_i^2}{\bar{h}^2}\, (\bar{h} + \rho^2 \sigma_Z^2 + \bar{q}) - h_i.$$

Using that $E_1\left[u_i^2\right] = Var_1(u_i) + E_1[u_i]^2$ (from the definition of variance), we get:

$$E_1\left[u_i^2\right] = h_i^2\,\frac{1}{\bar{h}^2}\,\left(\rho^2\left(\sigma_Z^2 + (\mu_Z - \bar{\gamma})^2\right) + \bar{h} + \bar{q}\right) - h_i = A\,h_i^2 - h_i.$$

Consequently, we get that $E_1\left[z_i^2\right] = \frac{1}{h_i}\,E_1\left[u_i^2\right] = A\,(h_0 + q_i) - 1$, such that the time-1 objective function (11) is given by:

$$E_1\left[E_2[C_i] - \frac{\rho}{2}\,Var_2(C_i)\right] = R_f\left(W_0 - \frac{\kappa(q_i)}{R_f}\right) + \frac{1}{2\,\rho}\,\left(A\,(h_0 + q_i) - 1\right). \qquad \text{(A9)}$$

Re-arranging the first-order condition of (A9) with respect to q_i (taking $\bar{q}$ and, hence, $\bar{h}$ and A as given), yields Theorem 3. In particular, while the right-hand side of the equation is constant, the left-hand is increasing in q_i (because κ is convex), so that an unique interior solution exists if $2\kappa'(0) < A/\rho$. Otherwise, one arrives at a corner solution: $q_i = 0$.

Finally, using the definition of A in (12), we get:

$$\frac{\partial A}{\partial \bar{\gamma}} = -\,\frac{2\,\rho^2}{\bar{h}}\,(\mu_Z - \bar{\gamma}) < 0, \quad \text{if } \mu_Z - \bar{\gamma} > 0. \qquad \text{(A10)}$$

One can also show that the value of information is declining in average aggregate precision $\bar{h}$:

$$\frac{\partial A}{\partial \bar{h}} = -\frac{2}{\bar{h}^3}\,\left(\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + \bar{h} + \bar{q}\right) + \frac{1}{\bar{h}^2} = \underbrace{-\frac{1}{\bar{h}^3}}_{<0}\,\underbrace{\left\{2\left(\rho^2\left(\sigma_Z^2 + \mu_Z^2\right) + \bar{q}\right) + \bar{h}\right\}}_{>0} < 0, \qquad \text{(A11)}$$

and, consequently, also declines in the average precision of private information $\bar{q}$:

$$\frac{\partial A}{\partial \bar{q}} = \frac{\partial A}{\partial \bar{h}}\,\frac{\partial \bar{h}}{\partial \bar{q}} = \underbrace{\frac{\partial A}{\partial \bar{h}}}_{<0}\,\left(\frac{2\,\bar{q}}{\rho^2\,\sigma_Z^2} + 1\right) < 0. \qquad \text{(A12)}$$

This result is reminiscent of the result on strategic substitutability in Grossman and Stiglitz (1980).

Proof of Theorem 4

In the information choice period, the objective of an investor with CARA expected utility is given by:

$$
\begin{aligned}
E_1\left[-E_2\left[\exp\left(-\rho\, C_i\right)\right]\right] &= E_1\left[-\exp\left(E_2\left[-\rho\, C_i\right] + \frac{1}{2} Var_2\left(-\rho\, C_i\right)\right)\right] \\
&= -\exp\left(\rho\, R_f\,\left(\frac{\kappa(q_i)}{R_f} - W_0\right)\right)\, E_1\left[\exp\left(-\frac{1}{2}\, z_i^2\right)\right], \qquad \text{(A13)}
\end{aligned}
$$

where we used (A6) and (A7) for the mean and variance of compensation C_i.

To compute the expectation of the exponential of a squared normal variable (z_i^2), we can use Brunnermeier (2001, page 64):

$$
\begin{aligned}
E_1\left[\exp\left(-\frac{1}{2}\, z_i^2\right)\right] &= \left|1 - 2\, Var_1(z_i)\,\left(-\frac{1}{2}\right)\right|^{-\frac{1}{2}} \times \\
&\quad \exp\underbrace{\left(\frac{1}{2}\left(-E_1[z_i]\right)^2 \left(1 - 2\, Var_1(z_i)\,\left(-\frac{1}{2}\right)\right)^{-1} Var_1(z_i) - \frac{1}{2}\, E_1[z_i]^2\right)}_{=\frac{1}{2}\, E_1[z_i]^2 \left(\frac{Var_1(z_i)}{1+Var_1(z_i)} - 1\right) = \frac{1}{2}\, E_1[z_i]^2 \left(\frac{Var_1(z_i) - 1 - Var_1(z_i)}{1+Var_1(z_i)}\right)} \\
&= \left(\left(1 + Var_1(z_i)\right)\, \exp\left(\frac{E_1[z_i]^2}{1 + Var_1(z_i)}\right)\right)^{-\frac{1}{2}}.
\end{aligned}
$$

One can further simplify the two components within the exponential function as follows:

$$
\begin{aligned}
1 + Var_1(z_i) &= 1 + \frac{Var_1(u_i)}{h_i} = h_i\, \frac{1}{\bar{h}^2}\left(\bar{h} + \rho^2 \sigma_Z^2 + \bar{q}\right) \equiv A_1\, h_i, \\
\frac{E_1[z_i]^2}{1 + Var_1(z_i)} &= \frac{h_i}{h_i + Var_1(u_i)}\, \frac{1}{h_i}\, E_1\left[u_i\right]^2 = \left(\bar{h} + \rho^2 \sigma_Z^2 + \bar{q}\right)^{-1} \rho^2\, \left(\mu_Z - \bar{\gamma}\right)^2 \equiv A_2.
\end{aligned}
$$

Consequently, the CARA objective function (A13) can be written as:

$$
\max_{q_i}\, E_1\left[-\exp\left(-\rho\, C_i\right)\right] = -\exp\left(\rho\, R_f\,\left(\frac{\kappa(q_i)}{R_f} - W_0\right)\right)\,\left(A_1\,\left(h_0 + q_i\right)\, \exp\left(A_2\right)\right)^{-\frac{1}{2}}.
$$

Note that neither $A_1\left(\bar{q}\right)$ nor $A_2\left(\bar{q}, \bar{\gamma}\right)$ depend on q_i. Hence, taking the first-order condition with respect to q_i and re-arranging yields Theorem 4.

Proof of Theorem 5

In equilibrium, precision choices are *mutual* best response functions. In particular, each investor's information choice q_i affects $\bar{q}$ and $\bar{q}$ affects q_i through A. Therefore, the equilibrium value of the precision of the private signal of the average investor, $\bar{q}$, is determined by plugging q_i from (12) into the definition (9) which yields (13).

To prove that the equilibrium is unique (within the class of linear equilibrium), it suffices to show that $\bar{q}$ is uniquely defined. Let $\Sigma(\bar{q}) = \frac{1}{\bar{q}} \int_0^1 q_i(\bar{q})\, di \geq 0$. Then $\bar{q}$ is defined as the solution of $\Sigma(\bar{q}) = 1$. Differentiating $\Sigma(\bar{q})$ yields:

$$\Sigma'(\bar{q}) = \underbrace{\frac{-1}{\bar{q}^2} \int_0^1 q_i(\bar{q})\, di}_{= -\frac{1}{\bar{q}} \Sigma(\bar{q}) < 0} + \underbrace{\frac{1}{\bar{q}}}_{>0} \int_0^1 \frac{\partial q_i}{\partial \bar{q}}\, di. \tag{A14}$$

Differentiating (12) with respect to $\bar{q}$ yields:

$$2\, \kappa''(q_i) \frac{\partial q_i}{\partial \bar{q}} = \frac{1}{\rho} \frac{\partial A}{\partial \bar{q}} \quad \Leftrightarrow \quad \frac{\partial q_i}{\partial \bar{q}} = \underbrace{\frac{1}{\rho\, 2\, \kappa''(q_i)}}_{>0} \underbrace{\frac{\partial A}{\partial \bar{q}}}_{<0 \text{ (see (A12))}} < 0,$$

where we used that κ is strictly convex. Consequently, $\Sigma'(\bar{q})$ is negative and $\Sigma(\bar{q})$ is decreasing over the real positive line. Together with $\Sigma(0) = +\infty$ and $\Sigma(\infty) = 0$, this implies that $\Sigma(\bar{q})$ "crosses" each real point once. Hence, there is a unique $\bar{q}$ satisfying $\Sigma(\bar{q}) = 1$ and, hence, (13).

Taking the derivative of (13) with respect to $\bar{\gamma}$, yields:

$$\frac{d\bar{q}}{d\bar{\gamma}} = \int_0^1 \frac{\partial q_i}{\partial \bar{\gamma}}\, di. \tag{A15}$$

Moreover, the derivative of the best information response (12) with respect to $\bar{\gamma}$ is:

$$2 \underbrace{\frac{\partial \kappa'}{\partial q_i}}_{= \kappa''(q_i)} \frac{\partial q_i}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} = \frac{1}{\rho} \underbrace{\left(\frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{\bar{\bar{q}}\bar{\gamma}} \right)}_{\frac{dA}{d\bar{\gamma}}} \Leftrightarrow \frac{\partial q_i}{\partial \bar{\gamma}} = \frac{1}{2\, \rho\, \kappa''(q_i)} \left(\frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{\bar{q}\bar{\gamma}} \right).$$

Substituting this expression into (A15) and simplifying, yields:

$$\frac{d\bar{q}}{d\bar{\gamma}} = \int_0^1 \frac{1}{2\,\rho\,\kappa''(q_i)} \left(\frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{\bar{q}\bar{\gamma}} \right) di = \underbrace{\int_0^1 \frac{1}{2\,\rho\,\kappa''(q_i)}\, di}_{\equiv H} \left(\frac{\partial A}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial A}{\partial \bar{q}} \frac{d\bar{q}}{d\bar{\gamma}} \right)$$

$$\Leftrightarrow \left(1 - \frac{\partial A}{\partial \bar{q}} H\right) \frac{d\bar{q}}{d\bar{\gamma}} = H \frac{\partial A}{\partial \bar{\gamma}} \quad \Leftrightarrow \quad \frac{d\bar{q}}{d\bar{\gamma}} = \underbrace{\left(1 - \frac{\partial A}{\partial \bar{q}} H\right)^{-1}}_{>0 \text{ because } \frac{\partial A}{\partial \bar{q}} < 0 \,(\text{see (A12)})} \quad H \quad \underbrace{\frac{\partial A}{\partial \bar{\gamma}}}_{<0\,(\text{see (A10)})} < 0.$$

Proof of Theorem 6

Taking the time-1 expectation of the equilibrium price (7) yields:

$$S = \frac{1}{\bar{h}} \left(\frac{\mu_X}{\sigma_X^2} + \rho\,\bar{\gamma} + \frac{\mu_Z\,\bar{q}}{\sigma_Z^2\,\rho} \right) + \frac{1}{\bar{h}} \left(\bar{h} - \frac{1}{\sigma_X^2} \right) \mu_X - \frac{1}{\bar{h}} \left(\frac{\bar{q}}{\rho\,\sigma_Z^2} + \rho \right) \mu_Z = \mu_X - \frac{1}{\bar{h}}\,\rho\,(\mu_Z - \bar{\gamma}).$$

Hence, the total derivative with respect to $\bar{\gamma}$ is given by:

$$\frac{dS}{d\bar{\gamma}} = \frac{\partial S}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial S}{\partial \bar{h}} \frac{d\bar{h}}{d\bar{\gamma}} = -\frac{1}{\bar{h}}\,\rho\,(-1) - \frac{-1}{\bar{h}^2}\,\rho\,(\mu_Z - \bar{\gamma})\,\frac{d\bar{h}}{d\bar{\gamma}} = \frac{1}{\bar{h}}\,\rho + \frac{1}{\bar{h}^2}\,\rho\,(\mu_Z - \bar{\gamma})\,\frac{d\bar{h}}{d\bar{\gamma}}.$$

Accordingly, the time-1 expectation of the excess return $M \equiv E_1[X - PR_f]$ is given by:

$$M = E_1[X] - E_1[PR_f] = \mu_X - \left(\mu_X - \frac{1}{\bar{h}}\,\rho\,(\mu_Z - \bar{\gamma}) \right) = \frac{\rho}{\bar{h}}\,(\mu_Z - \bar{\gamma})\,, \tag{A16}$$

and, thus, its total derivative with respect to $\bar{\gamma}$ is:

$$\frac{dM}{d\bar{\gamma}} = \frac{\partial M}{\partial \bar{\gamma}} \frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial M}{\partial \bar{h}} \frac{d\bar{h}}{d\bar{\gamma}} = \frac{-\rho}{\bar{h}} + \frac{-\rho}{\bar{h}^2}\,(\mu_Z - \bar{\gamma})\,\frac{d\bar{h}}{d\bar{\gamma}}.$$

Proof of Theorem 7

The unconditional variance V^2 is given by:

$$V^2 \equiv Var_1(X - PR_f) = E_1\left[(X - PR_f)^2\right] - E_1\left[(X - PR_f)\right]^2 = A - M^2,$$

where we used the law of iterated expectations, to re-write the first part as:

$$E_1\left[E_2\left[(X-PR_f)^2\right]\right] = \underbrace{E_1\left[Var_2(X)\right]}_{=Var_2(X)=\frac{1}{h_i}} + \underbrace{E_1\left[\left(\hat{\mu}_{X,i}-PR_f\right)^2\right)\right]}_{=E_1[z_i^2]\,(1/h_i)} = \frac{1}{h_i} + \frac{1}{h_i}\,(A\,h_i - 1) = A.$$

Accordingly, the unconditional variance is given by:

$$V^2 = \frac{1}{\bar{h}^2}\,\left(\rho^2\left(\sigma_Z^2+\mu_Z^2\right)+\bar{h}+\bar{q}\right) - \frac{\rho^2}{\bar{h}^2}\,(\mu_Z-\bar{\gamma})^2 = \frac{1}{\bar{h}^2}\,\left(\rho^2\sigma_Z^2+\bar{h}+\bar{q}\right),$$

which does not explicitly depend on $\bar{\gamma}$, but implicitly (through $\bar{q}$ in $\bar{h}$):

$$\frac{dV^2}{d\bar{\gamma}} = \frac{\partial V^2}{\partial\bar{\gamma}}\,\frac{d\bar{\gamma}}{d\bar{\gamma}} + \frac{\partial V^2}{\partial\bar{h}}\,\frac{d\bar{h}}{d\bar{\gamma}} = \frac{-1}{\bar{h}^3}\,\frac{d\bar{h}}{d\bar{\gamma}}.$$

B Proofs and Derivations for Section 3

Proof of Lemma 1

The first- and second-order derivatives of the CRRA-utility function (14) with respect to wealth W_i are given by:

$$\frac{\partial C_i^{1-\rho}/(1-\rho)}{\partial W_i} = C_i^{-\rho}, \qquad \frac{\partial^2 C_i^{1-\rho}/(1-\rho)}{\partial W_i^2} = -\rho\,C_i^{-\rho-1}.$$

As a result, the local coefficient of relative risk-aversion, denoted by $\hat{\rho}_i$, is given by:

$$\hat{\rho}_i = \frac{(-W_i)\,(-\rho)\,C_i^{-\rho-1}}{C_i^{-\rho}} = \rho\,\frac{W_i}{C_i} = \rho\,\frac{W_i}{W_i-\gamma_i W_{0,i}R_B-\kappa(q_i)},$$

which yields Lemma 1.

Derivations for the Approximate Portfolio Holdings

Substituting wealth $W_i = W_{0,i}\,(R_f + \phi_i\, r^e)$ into compensation C_i, we get:

$$\begin{aligned} C_i &= W_{0,i}(R_f + \phi_i r^e) - \gamma_i W_{0,i} R_B - \kappa(q_i) \\ &= (1-\gamma_i)\underbrace{\left(\frac{1}{1-\gamma_i} W_{0,i}\big(R_f + \phi_i r^e - \gamma_i R_B\big) - \frac{\kappa(q_i)}{1-\gamma_i}\right)}_{\equiv \tilde{C}_i}. \end{aligned} \tag{B1}$$

Importantly, with CRRA preferences, maximizing utility over C_i is equivalent to maximizing utility over $\tilde{C}_i$ because any multiplicative constant drops out of the optimization problem.

If the mean of $\tilde{C}_i$ is around 1, one can approximate $\tilde{C}_i$ using the following log-normal distribution:

$$\tilde{C}_i \approx \exp\left(-1 + E_1[\tilde{C}_i] - \frac{1}{2} Var_1(\tilde{C}_i) + \sqrt{Var_1(\tilde{C}_i)}\,\nu\right), \quad \nu \sim \mathcal{N}(0,1).$$

Because of log-normality of the approximated $\tilde{C}_i$, the time-2 portfolio choice problem with CRRA preferences can then be written as:

$$\begin{aligned} &\max_{\phi_1} \frac{1}{1-\rho} \exp\left((1-\rho)\left(-1 + E_1[\tilde{C}_i] - \frac{1}{2} Var_1(\tilde{C}_i)\right) + \frac{1}{2}(1-\rho)^2 Var_1(\tilde{C}_i)\right) \\ \Leftrightarrow &\max_{\phi_1} E_1[\tilde{C}_i] - \frac{\rho}{2} Var_1(\tilde{C}_i). \end{aligned}$$

Computing the mean and variance of $\tilde{C}_i$, taking the first-order condition with respect to ϕ_i and re-arranging, yields (16).

Numerical Solution Approach

Before describing our novel numerical solution approach for solving rational expectations equilibrium models, we first derive the investors' first-order conditions associated with their optimal portfolio and information choice—for the case of CRRA preferences and a single risky stock. These conditions form the basis of the numerical approach.

In period 2—the portfolio choice period—an investor (with arbitrary benchmarking concerns γ_i) chooses, conditional on his posterior beliefs, the fraction of wealth to be invested into the stock market, ϕ_i, in order to maximize his expected time-2 utility in (3) with $u_2(x) =$

$x^{1-\rho}/(1-\rho)$ subject to the "budget equation" $C_i = W_{0,i}(R_f + \phi_i\, r^e) - \gamma_i W_{0,i} R_B - \kappa(q_i)$. Computing the derivative with respect to ϕ_i, yields the following first-order condition:

$$E_2\big[C_i^{-\rho}\, r^e \,\big|\, Y_i, P\big] = 0, \tag{B2}$$

where $r^e \equiv X/P - R_f$ denotes the stock's excess return.

In period 1—the information choice period—each investor then chooses the precision of his private signal, q_i, in order to maximize his expected time-1 utility in (4) with $u_1(x) = x$. Computing the derivative with respect to q_i, yields the following first-order condition:

$$E_1\left[\frac{\partial U_{2,i}}{\partial q_i}\right] = 0. \tag{B3}$$

Together with the market-clearing condition for the stock, the first-order condition (B2)—for the different investors—characterizes the equilibrium in period 2. Importantly, note that the *stock price plays a dual role*: (i) it clears the stock market, and (ii) it appears in the information set used to compute each investor's conditional expectation in (B2). The key difficulty is that, in contrast to CARA-normal frameworks, the equilibrium price function is nonlinear, and its specific functional form is unknown. As a result, one cannot explicitly compute the investors' posterior beliefs and, hence, no closed-form solution for the equilibrium exists. Accordingly, the model has to be solved numerically.

Bernardo and Judd (2000) propose a numerical solution approach that relies on conjecturing (parameterizing) the equilibrium price function as well as investors' demand functions using Hermite polynomials. Using the projection method (see also Judd (1992)), the conditional expectation in (B2) is then replaced with a finite number of integration conditions. Consequently, the first-order condition (B2) but also the market-clearing condition will only hold approximately ("ε-equilibrium").

In this paper, we take a different approach. We discretize the state space of the economy which allows us to explicitly compute the investors' posterior beliefs and to solve the investors' first-order conditions (B2) and the market-clearing condition exactly. Moreover, the approach allows for arbitrary price and demand functions, that is, we do not parameterize (conjecture) these functions in any form. The disadvantage of the approach is that one can only compute the equilibrium for points within the discretization space (though one can make the space between gridpoints arbitrarily narrow or can rely on interpolation between grindpoints). For the setup described in Section 3.1 discretization is straightforward: Because of its binomial distribution, the payoff X can only take on two values: X_L or X_H. Hence, we solely have to discretize the noisy supply Z, which we do by use of N_Z discretization points.

Conditional on investors' information choices, the system of equations for each discretization point $(X_o, Z_n), o \in \{L, H\}, n \in \{1, \ldots, N_Z\}$ is then composed of five equations: The first-order condition (B2) for each of the four "sub-groups" of investors (that is, for benchmarked and non-benchmarked investors and in each case for the two possible realizations of the signal $Y_i \in \{Y_L, Y_H\}$) plus one market-clearing condition. The five unknowns are the stock price $P(X_o, Z_n)$ and the stock demand of each group of investors and for each signal realization: $\phi_i^{\mathcal{BI}}(X_o, Z_n, Y_i)$ and $\phi_i^{\mathcal{NI}}(X_o, Z_n, Y_i)$; $i \in \{L, H\}$. For numerical convenience, we also treat the aggregate demand for the stock as a variable and add its definition to the equation system, thereby increasing the number of equations (and unknowns) to 6.

If the posterior probabilities characterizing the conditional expectations in the first-order conditions (B2) were "exogenous" (e.g., a function of investors' private signals only or given by some prior beliefs), one could easily solve this equation system numerically. However, as mentioned above, the key difficulty is that the investors' posterior beliefs depend on the stock price. The advantage of our discretization is that it allows us to compute the corresponding posterior probabilities explicitly and exactly. In particular, note that, because each investor is small, the equilibrium stock price is a function of the stock's payoff and its supply only: $P(X, Z)$. Consequently, if the stock price function, conditional on a given value for the payoff X, is invertible (which we numerically verify), each investor can back out the two combinations of the payoff and the noisy supply, denoted by $\{(\hat{X}_L, \hat{Z}_L), (\hat{X}_H, \hat{Z}_H)\}$, which are consistent with a given price $\hat{P}$ (given information choices).[41] Using the distribution of the noisy supply, an investor can then compute the posterior probabilities for the payoff X.

Note that each "conjectured" supply $\hat{Z}_j$, $j \in \{L, H\}$ is given by the aggregate demand in the economy at price $\hat{P}$—conditional on the payoff being $\hat{X}_j$. Hence, to arrive at $\hat{Z}_j$, one needs to solve the portfolio choice problem of all four sub-groups of investors at price $\hat{P}$ and then aggregate accordingly—with the likelihood of low and high signals depending

[41] To distinguish between the values of the payoff and the noisy supply at a given gridpoint (X_o, Z_n), $o \in \{L, H\}, n \in \{1, \ldots, N_Z\}$, and the two "conjectured" payoff-noise combinations $\{(\hat{X}_j, \hat{Z}_j)\}$, $j \in \{L, H\}$ used to compute the posterior beliefs, we denote the later ones by a "ˆ". Similarly, to distinguish between the equilibrium price P and an (arbitrary) price used to illustrate the learning, we denote the later by $\hat{P}$.

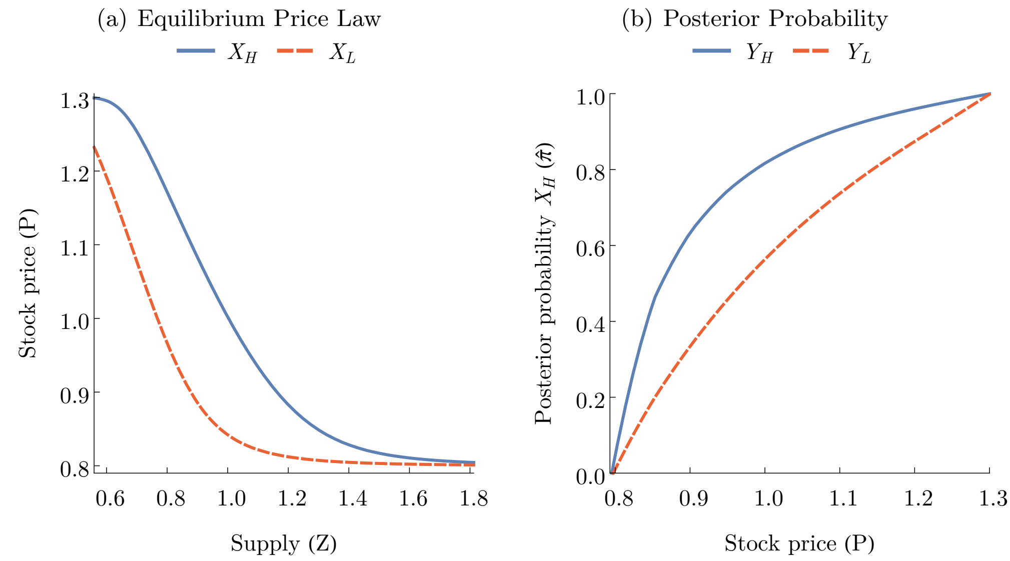

Figure A1: Numerical solution approach. Panel A shows the equilibrium price as a function of the noisy supply Z and the payoff X. Panel B shows an investor's posterior probability $\hat{\pi}_i$ as a function of the stock price, P—for the two possible signal realizations $Y_i \in \{Y_L, Y_H\}$. The graphs are based on the framework described in Section 3.1, with the following parameter values: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$ and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

on $\hat{X}_j$. Treating the aggregate demand as an unknown again, this adds $4+1$ equations for each $\hat{X}_j$, $j \in \{L, H\}$ to the system—in total $2 \times 5 = 10$ equations.

Formally, given the pair $\{(\hat{X}_L, \hat{Z}_L), (\hat{X}_H, \hat{Z}_H)\}$, investor i's posterior probability of the unobservable payoff X being high, conditional on price $\hat{P}$ and his private signal Y_i is:[42]

$$\hat{\pi}_i = \mathbb{P}(X_H \,|\, Y_i, \hat{P}) = \frac{f_Z(\hat{Z}_H)\, \mathbb{P}(X_H \,|\, Y_i)}{\sum_j f_Z(\hat{Z}_j)\, \mathbb{P}(X_j \,|\, Y_i)}, \quad j \in \{L, H\}; \tag{B4}$$

where $f_Z(\cdot)$ denotes the density function of the stochastic supply Z. The probabilities of the payoff X conditional on the private signal only, $\mathbb{P}(X_o \,|\, Y_i)$, can be computed using the correlation between an investor's signal and the payoff (implicit in the signal's precision q_i):

$$\mathbb{P}(X_o \,|\, Y_m) = \begin{cases} \frac{1}{2} + \frac{1}{2}\sqrt{\frac{q_i}{q_i+4}} & \text{if } o = m, \\ \frac{1}{2} - \frac{1}{2}\sqrt{\frac{q_i}{q_i+4}} & \text{if } o \neq m; \end{cases} \qquad \text{with } o, m \in \{L, H\}. \tag{B5}$$

[42] The posterior probability of the unobservable payoff X being low, $\mathbb{P}(X_L \,|\, Y_i, \hat{P})$, is simply given by $1 - \hat{\pi}_i$.

Panel A of Figure A1 depicts the equilibrium price law for the setting discussed in Section 3 of the paper. As one can see, the equilibrium price law is highly nonlinear.[43] Panel B of Figure A1 illustrates an investor's corresponding posterior beliefs. In particular, it shows the posterior probability $\hat{\pi}_i$, defined in (B4), for different combinations of the equilibrium stock price P, and the investor's private signal $Y_i \in \{Y_L, Y_H\}$. Intuitively, the posterior probability of the unobservable payoff being high is increasing in the stock price and always higher for the case in which the investor receives a high private signal.

Adding the 10 equations required for computing the posterior probabilities to the system of equations composed of the 6 equations discussed above (4 'explicit' first-order conditions, one market clearing condition and one aggregate demand definition) yields a system of 16 equations for each discretization point (X_o, Z_n). However, note that, at each discretization point, many of the equations (and matching variables) are actually redundant. For example, one solves, for each subgroup, the exact same first-order condition associated with the optimal portfolio choice (B2) three times (once for the 'explicit' first-order condition, once to compute the aggregate demand $\hat{Z}_L$, and, similarly, once to compute $\hat{Z}_H$). Moreover, one of the aggregate demand definitions associated with computing the aggregate demand for $\hat{X}_L$ or $\hat{X}_H$ is equivalent to the aggregate demand definition associated with the explicit market-clearing condition for the discretization point X_o, $o \in \{L, H\}$. Taking this into account allows us to reduce the system to 7 equations for each discretization point—4 first-order conditions (B2), 1 market-clearing condition and 2 aggregate demand definitions (1 associated with the explicit market-clearing condition and 1 associated with the 'off-equilibrium' aggregate demand used to compute the posterior probabilities).

In a setting with exogenous signal precisions, one would be able to solve this equation systems separately for each discretization point. However, in the presence of endogenous information choices, the problem has to be solved globally because the period-1 precision choices $q_i, \forall i$ affect the investors' period-2 posterior beliefs and portfolio choices which, in turn, affect the investors' information choices. That is, we arrive at a *combined system for all discretization points*, with $(N_Z \times 2) \times 7$ equations.

Finally, in order to evaluate the first-order conditions associated with the investors' optimal information choice (B3), one needs to compute, for each discretization point, the

[43] Intuitively, the stock price is a decreasing function of the noisy supply and higher in the case of a high underlying payoff (which increases the likelihood of high signals).

partial derivatives of the investors' time-2 value functions $U_{2,i}$ with respect to q_i.[44] The derivatives can be computed numerically by computing the investors' time-2 value functions $U_{2,i}$ also for $q_i + \delta$ (with δ being small). Because investors are price takers (also in period 1), the equilibrium price and, hence, learning from the price are unaffected by the change of precision to $q_i + \delta$ (technically, $\hat{Z}_j, j \in \{L, H\}$ is unchanged). However, the probability of the payoff conditional on the private signal, given in (B5), changes, it is based on $q_i + \delta$ now. In summary, for each discretization point, one needs to solve for the optimal portfolio (and, hence, value function) of each of the four sub-groups using the "new" posterior probabilities (based on $q_i + \delta$). Neither the market-clearing condition nor the equations associated with the learning from the stock price are required. This results in additional $(N_Z \times 2) \times 4$ equations.

Together with the two first-order conditions (B3) which characterize the two groups' information choices, this yields an overall system of $(N_Z \times 2) \times (4 + 7) + 2$ equations, with the same number of unknowns. We solve this large-scale fixed-point problem using Mathematica. In particular, we rely on `FindRoot` which uses a damped version of the Newton-Raphson method together with finite difference approximations to compute the Hessian. To facilitate the computations, we first solve the portfolio choice equation systems (separately for each discretization point) in the absence of private information. The solution of these systems are then used as a starting point for solving the same systems with a non-zero but exogenous signal precision (again, separately for each discretization point) before we use these systems' solutions as a starting point for the full system of equations with endogenous information choice.

We find that the solution of the full system is quite accurate for $N_Z = 25$, that is, further increasing the number of discretization points hardly changes the solution.[45] For that choice, the full system consists of $25 \times 22 + 2 = 552$ equations which can be solved on an Intel Core i7-980 workstation in about 3 minutes.

The numerical solution approach is quite flexible and can be used to address a far broader range of noisy rational expectations equilibrium models. For example, it can easily handle frictions, constraints or other preferences. Such variations would simply lead to a differ-

[44] Also, equation (B3) requires to compute the time-1 expectation over all realizations of X and Z. Numerically, this expectation is replaced by the expectation over the discretization space. This can either be done through interpolation or by 're-scaling' the probabilities for the finite number of gridpoints (so that they sum up to 1.0). Currently, we follow the second approach.

[45] Moreover, we have used our numerical approach to replicate the binomial setup with CARA preferences that is solved in closed-form in Breon-Drish (2015).

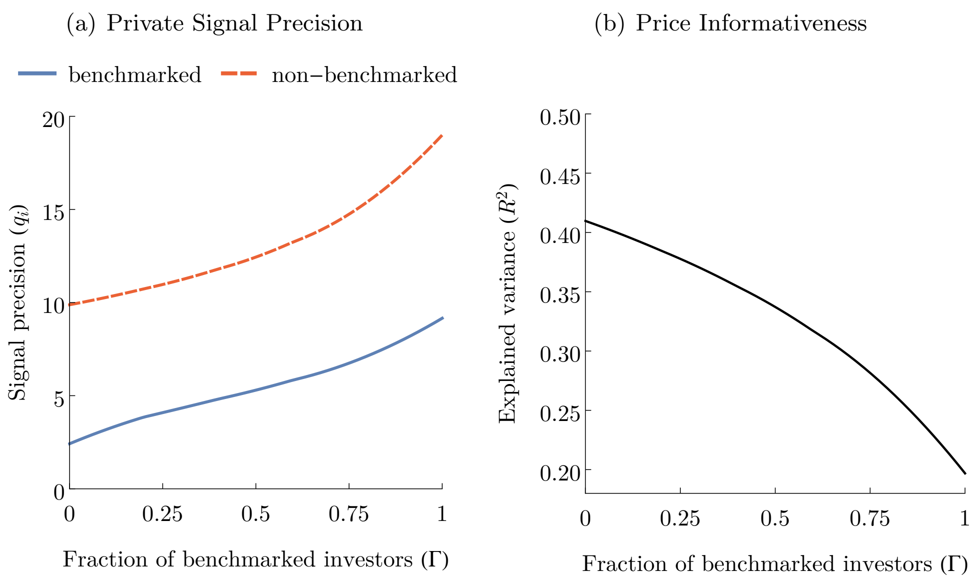

Figure A2: Equilibrium information choice with log-normal distributions. The figure illustrates equilibrium signal precisions, as a function of the fraction of benchmarked investors in the economy, Γ. Panel A shows the optimal signal precision of the two groups of investors and Panel B depicts the precision of the public price signal ("price informativeness"). Precision is measured as R^2, that is, the fraction of the variance of the payoff X that is explained by the investors' private information and the stock price, respectively. The graphs are based on the CRRA framework described in Section 3.1, but with log-normally distributed payoff and signals. The following parameter values are used: $\mu_X = 1.05$, $\sigma_X^2 = 0.25$, $\mu_Z = \ln(1) = 0$, $\sigma_Z^2 = 0.2$, $\rho = 3$, $\gamma_i = \gamma = 1/3\,\forall i \in \mathcal{BI}$, and an information cost function $\kappa(q_i) = \omega\, q_i^2$, with $\omega = 0.015$.

ent set of first-order conditions (B2) (plus—potentially—additional equations/constraints). Other than that, the algorithm would be unchanged. One can also extend the solution approach to a dynamic setting, as is done in Breugem (2018).

C Lognormally Distributed Payoff and Signals

The binomial distributions for the asset's payoff and the investors' private signals, as described in Section 3.1, are chosen for ease of exposition and numerical convenience only. The results also hold for the case of a log-normally distributed payoff and log-normally distributed signals.[46]

Figure A2 depicts the investors' optimal signal precisions and the resulting price informativeness for that case. Similar to the case with binomially distributed payoff and signals, the non-benchmarked investors endogenously chose a higher signal precision than the bench-

[46] In that case, one has to discretize the payoff space, e.g., by N_X gridpoints, which increases the overall number of discretization points to $N_X \times N_Z$. Moreover, one also has to discretize the private signals, e.g., by N_Y gridpoints, which yields $2 \times N_Y$ first-order conditions (B2)—one for each signal realization and group of investors.

marked investors. Moreover, price informativeness declines as the fraction of benchmarked investors increases. The results for the other quantities (omitted for brevity) are directly comparable to the setting with binomial distributions as well.

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