Economic Principles, Spring 2018
Final Exam

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You have two hours to complete this exam. Please answer the following three questions. Be sure to allocate your time in proportion to the points. Always justify your answers by providing a formal proof or a detailed argument. Good luck.

1. **[35 points]** Consider an economy with one consumer and one firm. There are two commodities. The consumer's initial endowment is $(e_1, e_2) = (45, 0)$, where e_j, $j = 1, 2$, denotes the endowment of commodity j.

   The firm produces the second commodity according to the production function $f\left(x_1^f\right) = 2\sqrt{x_1^f}$, where x_1^f denotes the amount of the first commodity purchased by the firm.

   The consumer's utility depends on his bundle (x_1, x_2) and the firm's production x_2^f. In particular, his preferences are given by:

$$u\left(x_1, x_2, x_2^f\right) = \ln x_1 + \ln x_2 - \frac{1}{2} \ln x_2^f$$

   a) Find the Pareto efficient allocation of this economy.

   b) Suppose the government introduces a per-unit tax τ on the firm's production (i.e., the firm pays τ on each unit of the second commodity). The government uses the tax revenues to make a transfer T to the consumer. Find the values of τ and T such that the Walrasian equilibrium allocation (of the economy with taxes and transfers) is Pareto efficient. Compute the Walrasian equilibrium prices of the two commodities (normalize $p_1 = 1$).

2. **[30 points]** A monopolist faces a consumer who has private information about his valuation of the good. A consumer with low valuation obtains the payoff $3q - x$ if he purchases the quantity q and makes a total payment of x. On the other hand, a consumer with high valuation obtains the payoff $4q - x$ if he purchases the quantity q and makes a total payment of x. Both types of consumers (l and h) obtain a payoff equal to zero if they do not trade with the monopolist. The two types of consumers are equally likely.

   The monopolist's cost of producing quantity q is q^2.

   The monopolist offers a menu with two contracts $((q_l, x_l), (q_h, x_h))$ (one for each type) to maximize his expected profits. Find the optimal menu.

3. [35 points] Consider the Rothschild-Stiglitz model of insurance. There are two groups of individuals. Low risk consumers incur a loss $L = 72$ with probability $\pi_l = \frac{19}{40}$. High risk consumers incur the loss $L = 72$ with probability $\pi_h = \frac{1}{2}$. Low and high risk consumers are identical in every other dimension. Their initial wealth is $W = 100$. Their von Neumann-Morgenstern utility function over positive levels of wealth is given by

$$u(x) = \sqrt{x}$$

Suppose that the fraction α of the high risk consumers is sufficiently large that the equilibrium exists.

Characterize the equilibrium of the model. In particular, let (W_1^i, W_2^i) denote the equilibrium contract of type $i = h, l$, where W_1^i represents the wealth in state 1 (no loss) and W_2^i represents the wealth in state 2 (there is a loss). Compute the values of W_1^h and W_2^h. Write the conditions that pin down the values of W_1^l and W_2^l.