

Collegio Carlo Alberto

Economic Principles Problem Set 1

1. Let \succsim be represented by $u : \mathbb{R}_+^n \rightarrow \mathbb{R}$. Prove that $u(x)$ is strictly quasiconcave if and only if \succsim is strictly convex.
2. Consider the following utility functions: $u(x_1, x_2) = \sqrt{x_1 x_2}$, and $v(x_1, x_2) = \ln(x_1) + \ln(x_2)$. Verify that u and v have the same indifference curves and the same MRS . Explain why.
3. Graph an indifference curve, and compute the MRS and the Marshallian demand functions for the following utility functions:
 - (a) Perfect substitutes: $u(x_1, x_2) = \alpha x_1 + \beta x_2$, where $\alpha > 0, \beta > 0$;
 - (b) Perfect complements: $u(x_1, x_2) = \min\{\alpha x_1, \beta x_2\}$, where $\alpha > 0, \beta > 0$.
4. (JR 1.21). We have noted that $u(x)$ is invariant to positive monotonic transformation. One common transformation is the *logarithmic transform*, $\ln(u(x))$. Take the logarithmic transform of the Cobb-Douglas utility function; then using that as the utility function, derive the Marshallian demand functions and verify that they are identical to those derived in class.
5. (JR 1.27). A consumer of two goods faces positive prices and has a positive income. Her utility function is

$$u(x_1, x_2) = \max\{ax_1, ax_2\} + \min\{x_1, x_2\}, \text{ where } 0 < a < 1.$$

Derive the Marshallian demand functions.

6. Consider a consumer with the following utility function:

$$u(x_1, x_2, x_3) = \sqrt{x_1 + 2x_2 + 3x_3}.$$

Denote by p_i , $i = 1, 2, 3$, the price of commodity i , and by y the consumer's income. Derive the Marshallian demand functions $x_i(p_1, p_2, p_3, y)$, $i = 1, 2, 3$.

Consider the following prices: $p_1 = 2$, $p_2 = 3$, $p_3 = 5$. Find the smallest income that allows the consumer to achieve a level of utility equal to 6.