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# Economic Principles
# Problem Set 10

**1.** Consider an economy with two consumers. The utility function of consumer $i = 1, 2$ is equal to

$$u(x^i, l^i, x^j) = \log(x^i + x^j) + l^i,$$

where $l^i$ denotes $i$'s (leisure) time and $x^i$ and $x^j$ denote the quantity of the public good purchased by consumer $i$ and $j$, respectively. Each consumer is endowed with 24 units of time and there is no endowment of the public good.

There is a firm that produces the public good. The firm has constant returns to scale and needs three units of time to produce one unit of the public good.

**(a)** Find the symmetric Walrasian equilibrium for this economy.

**(b)** Find the symmetric allocation that maximizes the utility of each consumer and show that it Pareto dominates the Walrasian equilibrium allocation.

**2.** Consider an economy with two consumers and two commodities. The first commodity is a private good while the second commodity is a pure public good. The utility function of consumer $i = 1, 2$ is $u^i(x^i, G)$, where $x^i$ denotes the quantity of private good consumed by $i$ and $G$ denotes the total quantity of the public good.

The initial total endowments of the private and public good are $T$ and zero, respectively. The private good can be consumed or can be used to produce the public good through the production function $G = f(x)$. Prove that any interior Pareto efficient allocation $(x^1, x^2, G)$ satisfies the following condition

$$\frac{\frac{\partial u^1(x^1,G)}{\partial G}}{\frac{\partial u^1(x^1,G)}{\partial x}} + \frac{\frac{\partial u^2(x^2,G)}{\partial G}}{\frac{\partial u^2(x^2,G)}{\partial x}} = \frac{1}{f'(T - x^1 - x^2)}$$

**3.** Consider a two-consumer, two-good exchange economy with externalities. Utility functions and endowments are:

$$u^A(x_1^A, x_2^A) = x_1^A + 2\ln(x_2^A) \quad \text{and} \quad \mathbf{e}^A = (12, 15),$$
$$u^B(x_1^B, x_2^B, x_2^A) = x_1^B + 2\ln(x_2^B) - \ln x_2^A \quad \text{and} \quad \mathbf{e}^B = (3, 5).$$

**(a)** Prove that if $x = \left(\left(x_1^A, x_2^A\right), \left(x_1^B, x_1^B\right)\right)$ is a Pareto efficient allocation with $x_1^A > 0$ and $x_1^B > 0$, then $x_2^A = \frac{20}{3}$ and $x_2^2 = \frac{40}{3}$.

**(b)** Find the Walrasian equilibrium prices and allocation. Is the equilibrium allocation Pareto efficient? Explain.

**(c)** Consider the efficient allocation $x^* = \left(\left(13, \frac{20}{3}\right), \left(2, \frac{40}{3}\right)\right)$. Suppose that the government can tax consumer $A$ with a (Pigouvian) per unit tax on consumption of good 2 . Furthermore, the government can redistribute revenue on a lump-sum basis in the amounts $T_i$, $i \in \{A, B\}$. Let $t$ denote the per-unit tax (i.e., consumer 2 pays $t$ for each unit of commodity 2 that he buys). Find $T_A^*$, $T_B^*$ and $t^*$ such that the Walrasian equilibrium allocation is $x^*$. Specify the Walrasian equilibrium prices. Is the government's budget constraint satisfied with equality? Explain.