

Collegio Carlo Alberto

Economic Principles Problem Set 11

1. A monopolist faces a market demand curve given by

$$q(p) = 70 - p.$$

- (a) If the monopolist can produce at constant average and marginal costs of $AC = MC = 6$, what output level will the monopolist choose to maximize profits? What is the price at this output level? What are the monopolist's profits?
- (b) Assume instead that the monopolist has a cost structure where total costs are described by

$$TC(q) = \frac{1}{4}q^2 - 5q + 300.$$

Find the price-quantity combination that maximize the monopolist's profits. What are the profits?

2. First-degree price discrimination.

Consider the following economy. There are two consumers, 1 and 2, who derive utility from a certain good (x) and from money (m). The consumers' utility functions are

$$\begin{aligned}u_1(x, m) &= 40\sqrt{x} + m, \\u_2(x, m) &= 60\sqrt{x} + m.\end{aligned}$$

A monopolist produces the good at constant average and marginal costs $c = 5$. Suppose that the monopolist observes the consumers' utility and makes take-it-or-leave-it offers (r_i, x_i) , $i = 1, 2$ (r_i denotes the payment that consumer i has to make to receive x_i units of the good). Compute the optimal offers (i.e. the offers that maximize the monopolist's profits).

3. Second-degree price discrimination.

Consider the economy described in Exercise 2. Suppose however that the monopolist does *not* observe the consumers' utility. Compute the optimal offers (r_i, x_i) , $i = 1, 2$. How does your answer differ from the answer to Exercise 2?

4. Third-degree price discrimination.

A monopolist sells in two markets. The demand curve for the monopolist's product is $q_1(p_1) = a_1 - b_1p_1$ in market 1, and $q_2(p_2) = a_2 - b_2p_2$ in market 2, where q_1 and q_2 are the quantities sold in each market, and p_1 and p_2 are the prices charged in each market. The monopolist has zero fixed and marginal costs. Note that although the monopolist can charge different prices in the two markets, it must sell all units within a market at the same price.

- (a) Under what conditions on the parameters (a_1, b_1, a_2, b_2) will the monopolist optimally choose not to price discriminate? (Assume interior solutions.)
 - (b) Now suppose that the demand functions take the form $q_i(p_i) = a_i p_i^{-b_i}$, for $i = 1, 2$, and the monopolist has some constant marginal cost of $c > 0$ (there are no fixed costs). Under what conditions will the monopolist choose not to price discriminate? (Assume interior solutions.)
5. (JR 6.1) Show that when the number of alternatives in X is restricted to just *two*, the method of majority voting does yield a social welfare relation that satisfies the conditions **U**, **WP**, **IIA**, and **D**.
6. Let there be three people in society, $\{1, 2, 3\}$, three alternatives, $\{x, y, z\}$, and let R^i denote individual i 's preference relation. Consider the social welfare function f defined as follows. Given any preference profile R^1, R^2, R^3 , $f(R^1, R^2, R^3)$ is the ordering induced by the majority rule if the ordering is transitive. If the ordering induced by the majority rule is not transitive, then $xIyIz$. Assume that the domain of f is unrestricted.
- Does f satisfy **WP**? Does f satisfy **D**? Does f satisfy **IIA**?