Collegio Carlo Alberto

Economic Principles
Problem Set 2

1. Bob consumes ice creams (x_1) and hamburgers (x_2). His utility function is

$$u(x_1, x_2) = (x_1)^{\frac{1}{2}} (x_2)^{\frac{1}{2}}.$$

   Bob's income is \$ 100. The price of each hamburger is \$ 2. The price of an ice cream depends on the quantity that Bob consumes. Specifically, he can buy the first ten ice creams at the price of \$ 2 each. For each additional ice cream there is a discount, and Bob has to pay only \$ 1.

   Derive Bob's budget constraint and compute his optimal consumption plan.

2. (JR 1.53). The n-good Cobb-Douglas utility function is:

$$u(x_1, x_2, ..., x_n) = A \prod_{i=1}^{n} x_i^{\alpha_i},$$

   where $A > 0$, $\alpha_1 > 0, ..., \alpha_n > 0$ and $\sum_{i=1}^{n} \alpha_i = 1$.

   (a) Derive the Marshallian demand functions.

   (b) Derive the indirect utility function.

   (c) Compute the expenditure function.

   (d) Compute the Hicksian demands.

3. Suppose that the utility function for two goods is given by

$$u(x_1, x_2) = \ln x_1 + x_2.$$

   Assume that $y > p_2$, and derive the Marshallian demand functions (the assumption guarantees that the consumer's problem has an interior solution). Verify that the cross-price effects do not coincide, i.e., $\frac{\partial x_1}{\partial p_2}$ is different from $\frac{\partial x_2}{\partial p_1}$.

4. An individual's utility function is

$$u(x_1, x_2, x_3) = g_1(x_1) + g_2(x_2) + g(x_3),$$

where $g_1(\cdot)$, $g_2(\cdot)$, and $g_3(\cdot)$ are all strictly concave continuous functions (so $g_i'(\cdot) > 0$, and $g_i''(\cdot) < 0$ for all $i = 1, 2, 3$). Show that all goods are normal.

5. (JR 1.50). Consider the utility function

$$u(x_1, x_2) = (x_1)^{\frac{1}{2}} + (x_2)^{\frac{1}{2}}.$$

(a) Compute the Marshallian demand functions, $x_i(p_1, p_2, y)$, $i = 1, 2$.

(b) Compute the substitution term in the Slutsky equation for the effects on x_1 of changes in p_2.

(c) Classify x_1 and x_2 as gross complements or substitutes.

6. John derives utility from wine (x_1) and beer (x_2). His utility function is

$$u(x_1, x_2) = \sqrt{(x_1)^2 + (x_2)^2},$$

and his income is \$ 300. In period 0 the price of wine and beer are $p_1 = 15$ and $p_2 = 5$, respectively. In period 1 the price of wine increases by \$ 5, i.e., $p_1' = 20$ (the price of beer does not change). Find the income which allows John to obtain in period 1 the same level of utility as in period 0.