Collegio Carlo Alberto

Economic Principles
Problem Set 3

1. (JR 2.8). The consumer buys bundle x^i at price p^i, $i = 0, 1$. Separately for parts (a) to (d), state whether these indicated choices satisfy WARP:

   (a) $p^0 = (1, 3)$, $x^0 = (4, 2)$; $p^1 = (3, 5)$, $x^1 = (3, 1)$.

   (b) $p^0 = (1, 6)$, $x^0 = (10, 5)$; $p^1 = (3, 5)$, $x^1 = (8, 4)$.

   (c) $p^0 = (1, 2)$, $x^0 = (3, 1)$; $p^1 = (2, 2)$, $x^1 = (1, 2)$.

   (d) $p^0 = (2, 6)$, $x^0 = (20, 10)$; $p^1 = (3, 5)$, $x^1 = (18, 4)$.

2. Consider the set of outcomes $C = \{c_1, c_2, c_3\}$, and let $\mathcal{L}$ denote the set of simple lotteries over C. Suppose that the preference relation $\succcurlyeq$ over $\mathcal{L}$ satisfies the independence axiom, and that $c_1 \succcurlyeq c_2 \succcurlyeq c_3$. Show that $c_1 \succcurlyeq L \succcurlyeq c_3$ for every lottery L.

3. Suppose that $U : \mathcal{L} \to \mathbb{R}$ represents the preference relation $\succcurlyeq$. Show that if U has the expected utility form, then $\succcurlyeq$ satisfies the independence axiom.

4. Consider the following lotteries: (L_1) \$5000 for sure; (L_2) a $\frac{1}{10}$ chance of \$30,000 and a $\frac{89}{100}$ chance of \$5000 (and a $\frac{1}{100}$ chance of nothing); (L_3) a $\frac{11}{100}$ chance of \$5000 (and a $\frac{89}{100}$ chance of nothing); and (L_4) a $\frac{1}{10}$ chance of \$30,000 (and a $\frac{9}{10}$ chance of nothing). Are the preferences $L_1 \succ L_2$ and $L_4 \succ L_3$ consistent with the independence axiom? (Assume that the preference relation is continuous.)

5. Let $\mathcal{L}$ denote the set of simple lotteries over the set of outcomes $C = \{c_1, c_2, c_3, c_4\}$. Consider the von Neumann-Morgenstern utility function $U : \mathcal{L} \to \mathbb{R}$ defined by

$$U(p_1, p_2, p_3, p_4) = p_2 u_2 + p_3 u_3 + 4p_4,$$

   where u_i, $i = 2, 3$, is the utility of the lottery which gives outcomes c_i with certainty. Suppose that the lottery $L_1 = \left(0, \frac{1}{2}, \frac{1}{2}, 0\right)$ is indifferent to $L_2 = \left(\frac{1}{2}, 0, 0, \frac{1}{2}\right)$, and that $L_3 = \left(0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ is indifferent to $L_4 = \left(0, \frac{1}{6}, \frac{5}{6}, 0\right)$. Find the values of u_2 and u_3.

6. Let $\mathcal{L} = \left\{ (p_1, p_2) \in [0,1]^2 : p_1 + p_2 = 1 \right\}$ be the set of lotteries over two outcomes. The preference relation $\succcurlyeq$ over $\mathcal{L}$ is represented by the utility function $U : \mathcal{L} \to \mathbb{R}$ defined by
$$U(p_1, p_2) = 3(p_1)^2 + 4(p_2)^2 + 4\sqrt{3}p_1p_2.$$
Find the optimal lottery. Does $\succcurlyeq$ satisfy the independence axiom?