Collegio Carlo Alberto

Economic Principles
Problem Set 4

1. Consider an investor who has initial wealth w and has to decide how to invest it. There is a riskless asset with rate of return r. The risky asset can have any of the rates of return x_i with probability π_i, $i = 1, ..., n$. Denote by α the fraction of wealth that the investor puts into the risky asset. Write the investor's problem. Show that if the investor has constant relative risk aversion, then the fraction of wealth invested in the risky asset, α, does not change with w (that is, $\frac{d\alpha^*}{dw} = 0$, where α^* denotes the solution to the investor's problem).

2. Consider the investor's problem described in Exercise 1. Assume that the investor is risk neutral and solve her problem.

3. An investor's initial wealth is $\$30,000$. The riskless asset has a rate of return equal to 1.05. The rate of return of the risky asset is 1.03 with probability $\frac{33}{100}$, and 1.06 with probability $\frac{67}{100}$. The von Neumann-Morgenstern utility function is $\ln(\cdot)$. Find the amount of wealth that the investor puts into the risky asset.

4. An individual is seen to place an even-money $\$100,000$ bet on the Lakers to win the NBA championship. If that individual has a $\ln(\cdot)$ von Neumann-Morgenstern utility function and his current wealth is $\$1,000,000$, what is the minimum probability he must assign to the Lakers winning the championship?

5. An individual has wealth W. Her von-Neumann Morgenstern utility function over non-negative levels of wealth is $u(y) = y^\rho$, where $0 < \rho < 1$. The individual is offered the following bet. If she pays x, with probability $1/2$ she receives nothing and with probability $1/2$ she receives $x(1+s)$, where $s > 1$. How much will she bet (as function of s)?