

Collegio Carlo Alberto

Economic Principles Problem Set 5

1. Suppose a production function has the form

$$y = (\min \{\alpha x_1, \beta x_2\})^\gamma,$$

where $\alpha > 0$, $\beta > 0$, $\gamma > 0$. Carefully sketch the isoquant map for this technology. Discuss returns to scale as a function of γ .

2. To produce output y a firm needs a patent $P \in \{0, 1\}$, whose cost is K , and a variable input x . Suppose that a patent and x units of input are able to produce $\sqrt{2x}$ of output. Let w denote the price of x .

- Write down the firm production function.
- Compute the marginal cost and the average cost of y . What is the relationship between average and marginal cost? Explain.

3. The production function for some good is given by

$$y = 5x_1 + 27x_2.$$

Let w_1 and w_2 denote the prices of inputs 1 and 2, respectively. Derive the conditional input demands.

4. (JR 3.21) A real-valued function is called *superadditive* if $f(z^1 + z^2) \geq f(z^1) + f(z^2)$. Show that every cost function is superadditive in input prices. Use this to prove that the cost function is non decreasing in input prices.

5. Consider a firm with the cost function

$$c(y, w_1, w_2) = y^2(w_1 + w_2),$$

where w_i denotes the price of input i , $i = 1, 2$. Let p denote the output price. Derive the output supply function $y(p, w_1, w_2)$, and the input demand functions $x_i(p, w_1, w_2)$, $i = 1, 2$.

6. Consider a firm with production function

$$y = (x_1^\rho + x_2^\rho)^{\frac{1}{\beta}},$$

where $0 < \rho < 1$, and $\beta > 0$. For what values of ρ and β are there (i) increasing returns to scale; (ii) constant returns to scale; (iii) decreasing returns to scale?

Suppose that there are decreasing returns to scale. Find the long run cost function. Derive the output supply function and the input demand functions.