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Economic Principles
Problem Set 8

1. Consider the following Robinson Crusoe economy. Robinson the consumer is endowed with zero units of coconuts, x, and 24 hours of time, h, so that $\mathbf{e} = (0, 24)$. His preferences are defined over $\mathbb{R}^2_+$ and represented by $u(x, h) = x^{\frac{1}{3}} h^{\frac{2}{3}}$. Robinson the producer uses the consumer's labor services, l, to produce coconuts, y, according to the production function $y = \sqrt{l}$. The producer sells the coconuts to the consumer. All profits from the production and sale of coconuts are distributed to the consumer. Find the Walrasian equilibrium prices and allocation of this economy.

2. In the Robinson Crusoe economy described in Exercise 1, suppose that Robinson does not think about a market, but simply chooses to enjoy h hours of leisure and spend $24 - h$ hours collecting coconuts. What is his optimal choice of h? How many coconuts does he get? Compare your answer to the answer to Exercise 1.

3. Consider the following economy. There are two firms, firm 1 and firm 2. Firm 1 produces commodity 1 out of labor, l, according to the production function $y_1 = \sqrt{l}$. Firm 2 produces commodity 2 out of l according to the production function $y_2 = l$. There are two agents, A and B, with identical utility function $u(x_1, x_2, h) = x_1 x_2$, where, x_k, $k = 1, 2$, denotes commodity k, and h denotes the leisure time. Each consumer is endowed with 6 units of time. There is no initial endowment of any of the two commodities. Finally, both consumers own half of each firm. Compute the Walrasian equilibrium prices and allocation.

4. (JR 5.29) Consider an economy with production in which there are many goods produced by the production sector, but each firm produces only one of them. Suppose also that each firm's output is given by a differentiable production function, and that each consumer's utility function is differentiable as well. Assume that this economy is in a Walrasian equilibrium with strictly positive prices and that all consumer's marginal utilities (of consumption goods) and all firm's marginal products (of inputs) are also strictly positive.

   (a) Show that the MRS between any two consumption goods is the same for each consumer.

(b) Show that the MRTS between any two inputs is the same for every firm using those inputs.

5. (JR 5.17) Consider an exchange economy with two identical consumers. Their common utility function is $u(x_1, x_2) = x_1^{\alpha} x_2^{1-a}$, for $0 < \alpha < 1$. Society has 10 units of x_1 and 10 units of x_2. Find endowments $\mathbf{e}^1$ and $\mathbf{e}^2$, where $\mathbf{e}^1 \neq \mathbf{e}^2$, and Walrasian equilibrium prices that will support as a Walrasian equilibrium allocation the equal-division allocation giving both consumers the bundle $(5, 5)$.