

# Collegio Carlo Alberto

## Economic Principles

### Problem Set 9

1. Consider a two-consumer, two-good exchange economy. Utility functions and endowments are:

$$\begin{aligned} u^A(x_1, x_2) &= (x_1 x_2)^2 & \text{and } \mathbf{e}^A &= (15, 6), \\ u^B(x_1, x_2) &= \ln(x_1) + 2 \ln(x_2) & \text{and } \mathbf{e}^B &= (3, 8). \end{aligned}$$

- (a) Derive the set of Pareto efficient allocations as a function of consumption of good 1 by consumer  $A$ .
- (b1) Characterize the allocations  $(x^A, x^B)$  in the core of this economy.
- (b2) (Optional) Derive the set of core allocations as a function of consumption of good 1 by consumer  $A$  (to completely solve this point you might need a computer).
- (c) Find the Walrasian equilibrium prices and allocation.
- (d) Verify that the Walrasian equilibrium allocation you found in part (c) is in the core.
2. (JR 5.13) An exchange economy has two consumers with expenditure functions:

$$\begin{aligned} e^1(\mathbf{p}, u) &= [3(1.5)^2 p_1^2 p_2 \exp(u)]^{1/3}, \\ e^2(\mathbf{p}, u) &= [3(1.5)^2 p_1 p_2^2 \exp(u)]^{1/3}. \end{aligned}$$

Suppose that the initial endowments are  $\mathbf{e}^1 = (10, 0)$  and  $\mathbf{e}^2 = (0, 10)$ . Find the Walrasian equilibrium.

3. (JR 5.32) In an economy with two types of consumer, each type has the respective utility function and endowments:

$$\begin{aligned} u^{1q}(x_1, x_2) &= x_1 x_2 & \text{and } \mathbf{e}^1 &= (8, 2), \\ u^{2q}(x_1, x_2) &= x_1 x_2 & \text{and } \mathbf{e}^2 &= (2, 8). \end{aligned}$$

- (a) Draw an Edgeworth box for this economy when there is one consumer of each type.

- (b) Characterize as precisely as possible the set of allocations that are in the core of this two-consumer economy.
- (c) Show that the allocation giving  $\mathbf{x}^{11} = (4, 4)$  and  $\mathbf{x}^{21} = (6, 6)$  is in the core of the two-consumer economy.
- (d) Now replicate the economy once, so that there are two consumers of each type, for a total of four consumers in the economy. Show that the double copy of the previous allocation, giving  $\mathbf{x}^{11} = \mathbf{x}^{12} = (4, 4)$  and  $\mathbf{x}^{21} = \mathbf{x}^{22} = (6, 6)$ , is *not* in the core of the replicated economy.