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Economic Principles
Solutions to Problem Set 11

Question 1

(a)
The maximization problem of the monopolist is:

$$\max_{q} \quad (70-q)q - 6q$$

The F.O.C. is

$$70 - 2q^* = 6 \Longrightarrow q^* = 32$$

Also, the monopolistic price and profits are

$$\begin{aligned} p^* &= 70 - q^* = 38 \\ \pi^* &= q^*(p^* - AC) = 32 \times (38-6) = 1024 \end{aligned}$$

(b) From $TC = \frac{1}{4}q^2 - 5q + 300$, we get $MC(q) = \frac{1}{2}q - 5$.
Again, at the monopolist's optimum

$$\max_{q} \quad (70-q)q - (\frac{1}{4}q^2 - 5q + 300)$$

The F.O.C. is:

$$70 - 2q^* = \frac{1}{2}q^* - 5 \Longrightarrow q^* = 30$$

The monopolistic price and profits are

$$\begin{aligned} p^* &= 70 - q^* = 40 \\ \pi^* &= p^*q^* - TC(q^*) = 40 \times 30 - \left(\frac{1}{4} \times 30^2 - 5 \times 30 + 300\right) = 825 \end{aligned}$$

Question 2

Since the monopolist observes the consumer's utility, it will make a take-it-or-leave-it offer to consumer 1 such that

$$40\sqrt{x_1} + m - r_1 = 40\sqrt{0} + m \Longrightarrow r_1 = 40\sqrt{x_1}$$

Similarly, it will make an offer to consumer 2 such that $r_2 = 60\sqrt{x_2}$
The optimal offers are the solutions to the following problem

$$\max r_1 + r_2 - 5\left(x_1 + x_2\right) \quad \text{s.t. } r_1 = 40\sqrt{x_1}, r_2 = 60\sqrt{x_2}$$

Hence,

$$r_1^* = 160, x_1^* = 16, r_2^* = 360, x_2^* = 36$$

Question 3
At the optimal offer (r_1, x_1), consumer 1's rationality constraint is binding and she has no surplus. Hence, $r_1 = 40\sqrt{x_1}$.
Also, since the monopolist does not observe the consumer's utility, the following incentive constraint for consumer 2 has to hold:

$$60\sqrt{x_2} + m - r_2 \geq 60\sqrt{x_1} + m - r_1 \Longrightarrow 60\left(\sqrt{x_2} - \sqrt{x_1}\right) \geq r_2 - r_1$$

To maximize profits, the monopolist will charge r_2 such that $r_2 = 60\left(\sqrt{x_2} - \sqrt{x_1}\right) + r_1 = 60\sqrt{x_2} - 20\sqrt{x_1}$
The monopolist's problem is

$$\max r_1 - 5x_1 + r_2 - 5x_2 \;\; \text{s.t.} \;\;\; r_1 = 40\sqrt{x_1}, r_2 = 60\sqrt{x_2} - 20\sqrt{x_1}$$

F.O.C.:

$$\begin{aligned} \frac{10}{\sqrt{x_1}} &= 5 \\ \frac{30}{\sqrt{x_2}} &= 5 \end{aligned}$$

Hence, $x_1 = 4, r_1 = 80, x_2 = 36, r_2 = 320$. The optimal offers are $(80, 4)$ and $(320, 36)$. Compared to exercise 2, consumer 1 consumes less x in the case of second degree price discrimination and his surplus is still zero. However, consumer 2 enjoys positive surplus when the monopolist does not observe consumers' utility.

Question 4
(a) The monopolist solves:

$$\max_{q_1,q_2} (a_1 - b_1 p_1)p_1 + (a_2 - b_2 p_2)p_2$$

:
F.O.C.

$$a_i - 2b_i p_i = 0 \Longrightarrow p_i^* = \frac{a_i}{2b_i} \;\; \text{for } i = 1, 2$$

Hence, if $\frac{a_1}{b_1} = \frac{a_2}{b_2}$, then the monopolist optimally chooses not to price discriminate.
(b) Given the demand functions $q_i(p_i) = a_i p_i^{-b_i}$ the monopolist solves:

$$\max_{p_1,p_2} \sum_{i=1}^{2} a_i p_i^{-b_i+1} - c\sum_{i=1}^{2} a_i p_i^{-b_i}$$

F.O.C.

$$(b_i - 1)a_i p_i^{-b_i} = c a_i b_i p^{-b_i - 1} \Longrightarrow p_i^* = \frac{cb_i}{b_i - 1} \;\text{for } i = 1, 2$$

Hence, if $\frac{cb_1}{b_1-1} = \frac{cb_2}{b_2-1}$, i.e., $b_1 = b_2$, the monopolist will choose not to discriminate.

Question 5

We check that when there are only two alternatives the method of majority voting yields a social welfare relation that satisfies **U**, **WP**, **IIA** and **D**. Suppose there are N (N>2) individuals, with associated preference relations $R_1, R_2, \ldots, R_N$ defined over the two alternatives x and y. Let the social welfare relation induced by majority voting be represented by f.

(i) Since there are only two alternatives, there is no possibility of cycling, so that it is always the case that either x wins over y, or y wins over x, or there is a tie (in which case, we'll assume xIy). Therefore the domain of f is indeed unrestricted.

(ii) If every individual strictly prefers x to y, then majority voting will lead to x being chosen. Therefore f also satisfies **WP**.

(iii) Since there are only two alternatives, it follows that for every individual, if $\tilde{R}_i$ ranks x and y in the same way as R_i then the two preference relations are one and the same and therefore majority voting leads to the same outcomes in both cases. Thus **IIA** is trivially satisfied.

(iv) Finally note that no individual can be a dictator. To see this, suppose that individual k prefers x to y, i.e. xP_ky. If all other individuals prefer y to x, then yPx. This means that individual k is not a dictator.

Question 6

Again, we check whether the proposed welfare relation satisfies **WP**, **IIA** and **D**.

(i) f satisfies **WP**, because if everyone prefers x to y there can be no cycling of preferences and we will have xPy.

(ii) f satisfies **D,** by a similar argument to the one used in Qn 5 (iv). Suppose that individual k prefers x to y, i.e. xP_ky. If all other individuals prefer y to x and if there is no cycling, then yPx. This means that individual k is not a dictator.

(iii) f does not satisfy **IIA**. To see this, consider the following table:

R_1	R_2	R_3	$\tilde{R}_1$	$\tilde{R}_2$	$\tilde{R}_3$
x	y	z	x	y	x
y	z	x	y	z	z
z	x	y	z	x	y

Note that when preferences are given by R_1, R_2, R_3 then xIyIz since there is a cycle. Now under $\tilde{R}_1, \tilde{R}_2, \tilde{R}_3$ we have xPy, yPz and xPz, although the relative ranking of x and y has not changed for any individual. Thus f does not satisfy IIA.