Collegio Carlo Alberto

Economic Principles
Solutions to Problem Set 6

Question 1
We know that in the long run equilibrium, price will be equal to the minimum of the AC of the production for a representative firm, and it is equal to MC as well.

$$AC(q) = \frac{c(q)}{q} = \frac{a}{q} + b + cq$$

$$\frac{dAC(q)}{dq} = -\frac{a}{q^2} + c$$

Hence, at the minimum of AC,

$$\frac{dAC(q)}{dq} = 0 \Longrightarrow q = \sqrt{\frac{a}{c}}$$

The minimum of $AC = \frac{a}{\sqrt{\frac{a}{c}}} + b + c \cdot \sqrt{\frac{a}{c}} = b + 2\sqrt{ac}$
Hence, the long run equilibrium price is equal to $b + 2\sqrt{ac}$ and the quantity of output produced by each firm is $\sqrt{\frac{a}{c}}$.

Question 2
First, since the production function of a representative firm is $q = \min\{\sqrt{x_1}, \sqrt{x_2}\}$, the cost function is

$$c(w_1, w_2, y) = w_1 y^2 + w_2 y^2$$

Given that $w_1 = 4$, $w_2 = 3$,

$$\begin{aligned} TC(y) &= 7y^2 \\ MC(y) &= 14y \end{aligned}$$

Since each firm is behaving competitively, they take output price as given. The condition for profit maximization is:

$$p = MC \Longrightarrow p = 14y \Longrightarrow y = \frac{p}{14}$$

Hence, the short run industry supply function is :

$$q^S(p) = 7 \times \frac{p}{14} = \frac{p}{2}$$

Together with $q^D(p) = 48 - p$, we find the short run equilibrium price and quantity.

$$q^S(p) = q^D(p) \Longrightarrow \frac{p}{2} = 48 - p \Longrightarrow p = 32$$

The short run equilibrium price is 32 and equilibrium quantity is 16.
For each firm, the short run equilibrium quantity is $\frac{16}{7}$.
Hence, for each firm, profit $= 32 \times \frac{16}{7} - 7 \times \left(\frac{16}{7}\right)^2 = \frac{256}{7}$.

Question 3

Let w_1 and w_2 be the prices for inputs 1 and 2, respectively.
The production function $y = \sqrt{(x_1)^2 + (x_2)^2}$ implies that the isoquants are concave. Therefore, the firm will use only the input that has a lower price.
When $w_1 < w_2$, only x_1 will be used in the production and when $w_2 < w_1$, only x_2 will be used in the production.
Hence,

$$\begin{aligned} c(w_1, w_2, y) &= \min\{w_1, w_2\} \cdot y \\ MC(w_1, w_2, y) &= \min\{w_1, w_2\} \end{aligned}$$

The long run equilibrium price will be equal to $\min\{w_1, w_2\}$ and quantity will be $a - b\min\{w_1, w_2\}$.
The long run equilibrium number of firms in the market is indeterminate, since the MC function for each firm is constant in y.
For example, suppose that the equilibrium number of firms is N and that all firms produce the same quantity of output. Then this quantity is

$$\left(\frac{a - b\min\{w_1, w_2\}}{N}\right)$$

Question 4

$$q^D(p) = q^S(p) \Longrightarrow p = 20, q = 80$$

The equilibrium price is 20 and the equilibrium quantity is 80.
Suppose the government imposes a per unit tax t on production, then

$$\begin{aligned} q_t^S(p) &= 20 + 3(p - t) \\ q^D(p) &= 100 - p \end{aligned}$$

At the equilibrium,

$$q_t^S(p) = q^D(p) \Longrightarrow p = 20 + \frac{3}{4}t \text{ and } q = 80 - \frac{3}{4}t$$

$$\text{Tax collected} = (80 - \frac{3}{4}t)t = 308 \Longrightarrow t = 4$$

Suppose the government imposes a per unit tax $\tilde{t}$ on consumption, then

$$\begin{aligned} q_{\tilde{t}}^D(p) &= 100 - (p + \tilde{t}) \\ q^S(p) &= 20 + 3p \end{aligned}$$

At the equilibrium,

$$q_{\tilde{t}}^D(p) = q^S(p) \Longrightarrow p = 20 - \frac{\tilde{t}}{4} \text{ and } q = 80 - \frac{3}{4}\tilde{t}$$

$$\text{Tax collected} = (80 - \frac{3}{4}\tilde{t})\tilde{t} = 308 \Longrightarrow \tilde{t} = 4$$

The deadweight loss incurred in both cases is equal to

$$\frac{1}{2} \times 4 \times (80 - \left(80 - \frac{3}{4} \times 4\right)) = 6$$

So, if the government wants to minimize the deadweight loss, it will be indifferent between two taxes.