Collegio Carlo Alberto

Game Theory
Problem Set 2

1. Find all Nash equilibria of the following normal-form games.

a)

	L	R
U	$3, 4$	$-2, 6$
D	$0, 3$	$-5, 1$

b)

	L	R
U	$4, 5$	$3, 1$
D	$4, 0$	$0, 6$

c)

	L	C	R
U	$6, 6$	$1, 2$	$3, 3$
M	$2, 1$	$4, 7$	$4, 3$
D	$3, 4$	$2, 5$	$3, 9$

2. **(Divide the dollar)** Players 1 and 2 are bargaining over how to split one dollar. Both players simultaneously name the amounts they would like to have, $s_1 \geqslant 0$ and $s_2 \geqslant 0$. If $s_1 + s_2 \leqslant 1$, then the players receive the amounts they named; if $s_1 + s_2 > 1$, then both players receive zero. Find all pure-strategy Nash equilibria of this game. Are there equilibria in weakly dominated strategies? Explain.

3. Compute all Nash equilibria of the Rock-Scissors-Paper game.

	R	S	P
R	$0, 0$	$1, -1$	$-1, 1$
S	$-1, 1$	$0, 0$	$1, -1$
P	$1, -1$	$-1, 1$	$0, 0$

4. Let $G = (S_1, \ldots, S_n, u_1, \ldots, u_n)$ and $\tilde{G} = (S_1, \ldots, S_n, \tilde{u}_1, \ldots, \tilde{u}_n)$ be two normal-form games with the same number of players and the same set of actions for every player. Suppose that for every player $i = 1, \ldots, n$, there exist two numbers $A_i > 0$ and B_i such that $\tilde{u}_i(s) = A_i u_i(s) + B_i$ for every action profile s in $S_1 \times \ldots \times S_n$. Show that a strategy profile $\sigma = (\sigma_1, \ldots, \sigma_n)$ is a Nash equilibrium of G if and only if σ is a Nash equilibrium of $\tilde{G}$.

5. Suppose that the normal-form game G^1 is derived from $G = (S_1, \ldots, S_n, u_1, \ldots, u_n)$ by eliminating pure strategies that are strictly dominated in G. Show that a strategy profile $(\sigma_1, \ldots, \sigma_n)$ is a Nash equilibrium of G if and only if σ is a Nash equilibrium of G^1. (NOTE: if S_i^1 is a subset of S_i, then any probability distribution σ_i in $\Delta(S_i^1)$ may be identified with the probability distribution in $\Delta(S_i)$ that gives the same probabilities as σ_i to the pure strategies in S_i^1, and gives probability 0 to the pure strategies that are in S_i but not in S_i^1.)