Collegio Carlo Alberto

Game Theory
Problem Set 3

1. (Cournot oligopoly) Consider the following oligopoly model. There are n firms. Let q_i denote the quantity produced by firm i. The market clearing price, p, depends on the total output:

$$p(q_1, \ldots, q_n) = \begin{cases} a - b\sum_{i=1}^{n} q_i & \text{if } a - b\sum_{i=1}^{n} q_i > 0 \\ 0 & \text{otherwise,} \end{cases}$$

where $a > 0$, $b > 0$. The total cost of firm i from producing quantity q_i is $C_i(q_i) = cq_i$. That is, there are no fixed costs and the marginal cost is constant and equal to c. Assume that $c < a$. Suppose that all firms choose their quantities simultaneously.

Define strategies and payoffs for the players. Find the symmetric Nash equilibrium in pure strategies. What happens as n approaches infinity? What is the economic interpretation of the limit result?

2. (Cournot oligopoly) Consider the oligopoly model described in Exercise 1 and suppose that $n = 2$. Let each duopolist have constant average and marginal costs, but suppose that $0 < c_1 < c_2$. Show that firm 1 will have greater profits and produce a greater share of the market output than firm 2 in the Nash equilibrium.

3. (Bertrand oligopoly with homogeneous products) Two firms produce a homogeneous good. Each firm has constant marginal costs $c > 0$, and no fixed cost. The two firms choose their prices simultaneously. Let p_i denote the price of firm i. If p_i and p_j are the prices chosen by the firms, the quantity that consumers demand from firm i is:

$$q_i(p_i, p_j) = \begin{cases} a - bp_i & \text{if } p_i < p_j \\ \frac{1}{2}(a - bp_i) & \text{if } p_i = p_j \\ 0 & \text{if } p_i > p_j \end{cases},$$

where $a > bc$, $b > 0$. In other words, consumers buy only from the firm which charges the lowest price. If both firms charge the same price, they share the demand equally.

Define strategies and payoffs for the players. Find all pure strategies Nash equilibria.

4. **(Bertrand oligopoly)** In the model described in Exercise 3 firms have no fixed costs and identical marginal cost. We now analyze what happens when we relax these assumptions.

Find a Nash equilibrium pair of prices, (p_1, p_2), and quantities, (q_1, q_2), when the following hold.

(a) Firm 1 has fixed costs $F > 0$ (assume that firm 2 gets the entire market if both firms charge the same price).

(b) Both firms have fixed costs $F > 0$ (again, assume that firm 2 gets the entire market if both firms charge the same price).

(c) Fixed costs are zero, but firm 1 has lower marginal cost than firm 2, so $c_2 > c_1 > 0$. (For this one, assume the low-cost firm captures the entire market demand whenever the firms charge equal prices.)

5. **(Bertrand oligopoly)** Two firms produce a homogeneous good. Each firm has constant marginal costs $c > 0$, and no fixed cost.

In addition to choosing its price, a firm can submit a "meet the competition" clause. By submitting this clause, the firm promises to match the competitor's price if the latter is lower. In other words, the set of strategies of each firm is $\mathbb{R}_+ \times \{Y, N\}$, where Y (N) means that firm submits (does not submit) the clause. The two firms make their decisions simultaneously.

Market demand is $q = a - bp$, where $a > bc$, $b > 0$. Consumers buy only from the firm which charges the lowest price. If both firms charge the same price, then they share market demand equally.

Find all pure strategy Nash equilibria.

6. **(First price auction)** Consider the following auction. There are $n > 1$ bidders who compete for a prize $v > 0$. Bidders simultaneously submit their bids. The bidder with the highest bid gets the prize and pays her bid (if two or more players submit the highest bid, they are equally likely to get the prize and pay their bid). A bidder does not pay anything if she does not get the prize.

Formally describe this auction as a game, with strategy sets and payoff functions for the players. Describe all pure strategy Nash equilibria.