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Game Theory

Problem Set 5

1. **(Bertrand duopoly with incomplete information)** Consider the following model of Bertrand duopoly with *differentiated* products and asymmetric information. Demand for firm $i = 1, 2$ is equal to:

$$q_i\left(p_i, p_j\right) = \begin{cases} a - p_i - b_i p_j & \text{if } p_i \leqslant a - b_i p_j \\ 0 & \text{otherwise.} \end{cases}$$

Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm i, $b_i = b_L$ with probability θ and $b_i = b_H$ with probability $1 - \theta$, independent of the realization of b_j. Each firm knows its own b_i but not its competitor's. All of this is common knowledge. What are the action spaces, type spaces, beliefs, and utility functions in this game? What are the strategy spaces? What conditions define a symmetric pure-strategy Bayesian Nash equilibrium of this game? Solve for such an equilibrium.

2. **(A war game)** Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either "attack" or "not attack". In addition, each army is either "strong" or "weak" with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth $M = 10$ if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a "cost" of fighting, which is s if it is strong and w if it is weak, where $s = 6$ and $w = 8$. There is no cost of attacking if its rival does not.
Model this situation as a Bayesian game and identify all pure-strategy Bayesian Nash equilibria.

3. **(Is information always beneficial?)** Consider the following Bayesian game. Player 1 may be either type a or type b, where both types are equally likely. Player 2 has no private information. Depending on player 1's types, the payoffs to the two players depend on their actions in $A_1 = \{U, D\}$ and $A_2 = \{L, C, R\}$ as shown in the following table.

	$t_1 = a$		
	L	C	R
U	$4, -1$	$3, 0$	$3, -3$
D	$5, 4$	$2, 5$	$2, 0$

	$t_1 = b$		
	L	C	R
U	$4, -1$	$3, -3$	$3, 0$
D	$5, 4$	$2, 0$	$2, 5$

Show that this game has a unique Bayesian Nash equilibrium, in which player 2's expected payoff is higher than her payoff in the unique equilibrium of any of the related games in which she knows player 1's type.