Collegio Carlo Alberto

Game Theory
Problem Set 8

1. Let G be the following normal-form game:

	A	B	C
A	$5,5$	$0,6$	$0,0$
B	$6,0$	$3,3$	$0,0$
C	$0,0$	$0,0$	$1,1$

Consider all symmetric SPE of the repeated game in which the game G is repeated T times and each player's payoff is the sum of the payoffs obtained each period (there is no discounting). Let $\bar{u}(T)$ be the maximum average (per period) payoff of player 1 in any of these equilibria, and let $\underline{u}(T)$ be the corresponding minimum. Find $\bar{u}(T)$ and $\underline{u}(T)$.

2. Suppose the game G below is repeated twice. Each player's payoff is the discounted sum of the payoffs obtained in each period.

	A	B	C
A	$0,0$	$3,4$	$6,0$
B	$4,3$	$0,0$	$0,0$
C	$0,6$	$0,0$	$5,5$

Let δ be the discount factor. Find the values of δ for which there exists a SPE in which the action profile (C,C) is played in the first period.

3. (Gibbons, Exercise 2.10, page 134). The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable x is greater than 4, so that $(4,4)$ is not an equilibrium payoff in the one-shot game.

	P_2	Q_2	R_2	S_2
P_1	$2,2$	$x,0$	$-1,0$	$0,0$
Q_1	$0,x$	$4,4$	$-1,0$	$0,0$
R_1	$0,0$	$0,0$	$0,2$	$0,0$
S_1	$0,-1$	$0,-1$	$-1,-1$	$2,0$

For what values of x is the following strategy (played by both players) a subgame-perfect equilibrium?

Play Q_i in the first stage. If the first-stage outcome is (Q_1, Q_2), play P_i in the second stage. If the first-stage outcome is (y, Q_2) where $y \neq Q_1$, play R_i in the second stage. If the first-stage outcome is (Q_1, z) where $z \neq Q_2$, play S_i in the second stage. If the first-stage outcome is (y, z) where $y \neq Q_1$ and $z \neq Q_2$, play P_i in the second stage.

4. (Gibbons, Exercise 2.11, page 134). The simultaneous-move game (below) is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. Can the payoff $(4, 4)$ be achieved in the first stage in a pure-strategy subgame-perfect equilibrium? If so, give strategies that do so. If not, prove why not.

	L	C	R
T	$3, 1$	$0, 0$	$5, 0$
M	$2, 1$	$1, 2$	$3, 1$
B	$1, 2$	$0, 1$	$4, 4$