1. Let $G$ be the following normal-form game:

\[
\begin{array}{ccc}
A & B & C \\
A & 5,5 & 0,6 & 0,0 \\
B & 6,0 & 3,3 & 0,0 \\
C & 0,0 & 0,0 & 1,1 \\
\end{array}
\]

Consider all symmetric SPE of the repeated game in which the game $G$ is repeated $T$ times and each player’s payoff is the sum of the payoffs obtained each period (there is no discounting). Let $\bar{u}(T)$ be the maximum average (per period) payoff of player 1 in any of these equilibria, and let $\underline{u}(T)$ be the corresponding minimum. Find $\bar{u}(T)$ and $\underline{u}(T)$.

2. Suppose the game $G$ below is repeated twice. Each player’s payoff is the discounted sum of the payoffs obtained in each period.

\[
\begin{array}{ccc}
A & B & C \\
A & 0,0 & 3,4 & 6,0 \\
B & 4,3 & 0,0 & 0,0 \\
C & 0,6 & 0,0 & 5,5 \\
\end{array}
\]

Let $\delta$ be the discount factor. Find the values of $\delta$ for which there exists a SPE in which the action profile $(C, C)$ is played in the first period.

3. (Gibbons, Exercise 2.10, page 134). The accompanying simultaneous-move game is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. The variable $x$ is greater than 4, so that $(4, 4)$ is not an equilibrium payoff in the one-shot game.

\[
\begin{array}{ccccc}
P_1 & Q_2 & R_2 & S_2 \\
P_2 & 2,2 & x,0 & -1,0 & 0,0 \\
Q_1 & 0,x & 4,4 & -1,0 & 0,0 \\
R_1 & 0,0 & 0,0 & 0,2 & 0,0 \\
S_1 & 0,-1 & 0,-1 & -1,-1 & 2,0 \\
\end{array}
\]
For what values of $x$ is the following strategy (played by both players) a subgame-perfect equilibrium?

Play $Q_i$ in the first stage. If the first-stage outcome is $(Q_1, Q_2)$, play $P_i$ in the second stage. If the first-stage outcome is $(y, Q_2)$ where $y \neq Q_1$, play $R_i$ in the second stage. If the first-stage outcome is $(Q_1, z)$ where $z \neq Q_2$, play $S_i$ in the second stage. If the first-stage outcome is $(y, z)$ where $y \neq Q_1$ and $z \neq Q_2$, play $P_i$ in the second stage.

4. (Gibbons, Exercise 2.11, page 134). The simultaneous-move game (below) is played twice, with the outcome of the first stage observed before the second stage begins. There is no discounting. Can the payoff $(4,4)$ be achieved in the first stage in a pure-strategy subgame-perfect equilibrium? If so, give strategies that do so. If not, prove why not.

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