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Game Theory

Problem Set 9

1. Suppose there are n firms in a Cournot oligopoly. Inverse demand is given by:

$$P(q_1, \ldots, q_n) = a - (q_1 + \ldots + q_n).$$

For simplicity, assume that there are no production costs. Consider the infinitely repeated game based on this stage game. What is the lowest value of δ such that the firms can use trigger strategies to sustain the monopoly output level in a SPE? How does the answer vary with n, and why?

2. Consider an infinitely repeated game $G(\infty, \delta)$ in which the following "Game of Chicken" is the stage game G :

	A_2	B_2
A_1	$4,4$	$1,6$
B_1	$6,1$	$0,0$

a) Find the minmax value $\underline{v}_i$ of player $i = 1, 2$.

b) Define carefully the set of payoffs identified in the Fudenberg-Maskin Folk Theorem as SPE payoffs if δ is sufficiently large.

c) Consider the strategy profile s defined as follows: (i) at $t = 0$ player i plays A_i; (ii) in any period $t > 0$, player i plays A_i if (A_1, A_2) or (B_1, B_2) was played in the previous period; (iii) in any period $t > 0$, player i plays B_i if (A_1, B_2) or (B_1, A_2) was played in the previous period. Find the values of δ for which s is a SPE (use the principle of optimality).

3. Consider the following stage game G :

	A	D
A	$2,3$	$1,5$
D	$0,1$	$0,1$

Suppose that G is played infinitely many times and the discount factor is $\delta = 1/2$. Show that $((A, A), (A, A), (A, A), \ldots)$ is not a subgame perfect equilibrium outcome path of $G\left(\infty, \frac{1}{2}\right)$.

4. (Gibbons, Exercise 2.17, page 136). Consider the following infinite-horizon game between a single firm and a sequence of workers, each of whom lives for one period. In each period the worker chooses either to expend effort and so produce output y at effort cost c or to expend no effort, produce no output, and incur no cost. If output is produced, the firm owns it but can share it with the worker by paying a wage, as described next. Assume that at the beginning of the period the worker has an alternative opportunity worth zero (net of effort cost) and that the worker cannot be forced to accept a wage less than zero. Assume also that $y > c$ so that expending effort is efficient.

Within each period, the timing of events is as follows: first the worker chooses an effort level, then output is observed by both the firm and the worker, and finally the firm chooses a wage to pay the worker. Assume that no wage contracts can be enforced: the firm's choice of wage is completely unconstrained. In a one-period game, therefore, subgame-perfection implies that the firm will offer a wage zero independent of the worker's output, so the worker will not expend any effort.

Now consider the infinite-horizon problem. Recall that each worker lives for only one period. Assume, however, that at the beginning of period t, the history of the game through period $t - 1$ is observed by the worker who will work in period t. Suppose the firm discounts the future according to the discount factor δ per period. Describe strategies for the firm and each worker in a subgame-perfect equilibrium in the infinite horizon game in which in equilibrium each worker expends effort and so produces output y, provided the discount factor is high enough. Give a necessary and sufficient condition for your equilibrium to exist.